Optimal online management of V2G charging in corporate parking facilities ⋆

Davide Liuzza[a,*], Elisa Mostacciuolo[a], Valerio Mariani[b], Luigi Iannelli[a], Francesco Vasca[a], Giuseppe Silano[c]

[a]Department of Engineering, University of Sannio, Benevento, 82100, Italy,
[b]Department of Energy Technologies and Renewable Energy Sources, Enea , Portici, Napoli, 80055, Italy,
[c]Department of Power Generation Technologies and Materials, Ricerca sul Sistema Energetico (RSE) S.p.A. , Milano, 20135, Italy, Department of Cybernetics, Czech Technical University, Prague, Czech Republic,

Abstract

The widespread adoption of electric vehicles (EVs) requires a well-developed charging infrastructure and efficient management strategies. In this context, providing charging opportunities for employees during working hours presents a promising approach. This paper proposes a parking management system for EVs in a corporate setting, designed to optimize the allocation of vehicles to charging stations throughout the workday while managing the power flow to minimize costs and enhance participation in various energy markets. The system addresses real-world challenges, including asynchronous booking requests, stochastic variations in EVs usage, and disparities between the number of charging stations and the demand for charging typically with fewer stations than EVs requiring service. Additional practical constraints are considered, such as variations in charging station characteristics and employees' availability constraints, which may prevent them from plugging in or retrieving their vehicles at certain times. The proposed solution is designed as a service-oriented system capable, at the same time, of dynamically allocating and rescheduling charging sessions in response to user requests. Deterministic, such as confirmed bookings, and forecast data have been integrated in the proposed solution to improve operational efficiency.

1. Introduction

The broad diffusion of electric vehicles (EVs) has highlighted the critical need for efficient and accessible charging infrastructure [1]. Smart parking systems are emerging as a promising solution in the EV landscape, which allows integration of EV charging and the public infrastructure [2].

Vehicle-to-Grid (V2G) technology plays a transformative role by enabling bidirectional power exchange, turning EVs from passive consumers into active grid participants [3]. This capability allows EVs to support power grid stability by mitigating fluctuations and contributing to reserve services. However, effectively coordinating the EVs charging and discharging behaviour for optimal grid participation remains a key challenge [4].

Beyond technical advantages, V2G technology offers economic incentives and supports environmental and social goals through strategic participation in the electricity markets [5, 6, 7]. V2G systems reduce urban congestion, lower emissions, and promote sustainable energy practices, benefiting both EVs users and broader urban communities. Then optimizing the power management in charging stations and smart parking lots becomes crucial for research and development. Indeed, equipping parking lots with charging points transforms them into dual-function facilities, offering both parking and charging services, particularly beneficial for locations with extended parking hours, such as office buildings, shopping malls, and airports [8]. However, energy management within these systems is complex due to uncertainties such as arrival and departure times, initial state of charge (SOC), and variable energy consumption [9]. In [10] an optimization model that addresses the EV users' preferences in terms of economic and quality of service (QoS) dimensions is analysed.

Moreover, another challenge is related to the management of the congestion in parking lots due to a limited number of charging stations [11, 12]. Previous research has explored metaheuristic-based charging scheduling [13] to create a charging schedule for EVs with a reservation system, but this solution proved to be slow for real-time implementation. Furthermore, energy aggregators participating in wholesale markets must handle uncertainties in ancillary services while optimizing revenue through V2G operations [14, 15].In [16] the authors deal with a deep learning-based framework for day-ahead optimal charging scheduling while in [17] a day-ahead co-optimization algorithm to minimize energy losses and transformers operating costs simultaneously is proposed. This paper proposes a parking management system for a corporate setting with a limited number of charging stations, where employees charge their EVs during work hours. Users can book charging slots by specifying arrival and departure times, desired final charge level, and unavailable time slots (e.g., due to meetings or work commitments). The presented approach is based on a

⋆This work was partially funded by the Research Fund for the Italian Electrical System under the Three-Year Research Plan 2022-2024 (DM MITE n.337, 15.09.2022) and by the CTU grant no. SGS23/177/OHK3/3T/13.

*Corresponding author
Email addresses: davide.liuzza@unisannio.it (Davide Liuzza), emostac@unisannio.it (Elisa Mostacciuolo), valerio.mariani@enea.it (Valerio Mariani), luigi.iannelli@unisannio.it (Luigi Iannelli), vasca@unisannio.it (Francesco Vasca), giuseppe.silano@rse-web.it (Giuseppe Silano)

mixed-integer linear programming (MILP) model.

Recent studies have explored MILP models for smart parking and the integration charging stations with renewable energy and storage systems, some of them primarily focusing on component sizing [18, 19]. Earlier work has examined power management in public parking facilities with pre-allocated EVs [20]. Other MILP formulations address EVs aggregator policy [21] and EVs fleet charging from photovoltaic sources [22, 23] but differently from our study, they focus only on energy balance in the ancillary services market (ASM), overlooking technical constraints such as ramp rates and minimum delivery durations. Similarly, studies modeling EVs charging and discharging in solar-powered parking lots exclude participation in the day-ahead market and fail to consider stochastic EVs arrivals [24]. In [25] is presented a research on battery energy storage systems sizing for EVs parking lots which applies stochastic methods for demand estimation but does not incorporate real-time operational constraints. A V2G control strategy proposed in [26] optimizes EVs charging stations for ASM and day-ahead market participation but does not account for intraday market adjustments.

Our work extends the results presented in [27], where an optimization framework is proposed for day-ahead EVs charging scheduling in a corporate parking lot. We improve the framework by considering various uncertainties and enforcing participation in energy markets. The system's cost function prioritizes charging during optimal time slots and, at the same time, considers a scheduling problem for the (limited) recharging spots, while promoting market participation, ensuring each EVs reaches its desired SOC by the end of the workday. This is obtained by playing on the energy markets, by considering the day-ahead and the ancillary services. Unlike [27], our approach explicitly integrates infra-day market operations, enabling dynamic energy trading to optimize both charging costs and grid contributions.

Furthermore, we consider the possibility of a flexible quality of service, where employees can accept or refuse a final SOC that differs from their initial request. Our approach accounts for a realistic scenario where the number of charging stations is typically lower than the number of EVs request charging, inherently requiring a combined scheduling and power management solution. The system also supports heterogeneous charging stations with different technical characteristics.

Also, differently to the earlier works, our framework is designed for an on the field implementation as an online service. In such a setting, EVs may asynchronously request charging slots while others are already scheduled or expected to arrive without prior booking, introducing stochastic elements into the problem. Additionally, the system allows users to specify "do not disturb" intervals, ensuring their vehicles are not required to be plugged in or removed from charging stations during specific periods. According to this online approach in our paper we assume that an offline solution has already been computed. The calculation of an offline management trajectory on historical/forecasted data, usually a day in advance, has been addressed in several articles in the literature (see [27]). Our online approach exploits this day-ahead computed solution to provide both scheduling and power management in runtime on real data (commonly different from the forecasted data used for the offline solution). Our algorithm attempts to adhere as much as possible to the offline computed scheduling but taking into account real-time deterministic data and realistic operating constraints while, at the same time, being fast enough to be proposed as an online service. While in the first part of the paper (up to Section 5) the day-ahead offline solution is assumed known and only real-time deterministic data are considered for recomputing an online management, in the second part of the paper (Section 7 and Section 8) we extend our approach integrating forecasted data with deterministic ones. This extension is also useful for computing offline solutions the day before by powering the online management algorithm.

Finally our solution provides, in a unified approach, a more comprehensive, adaptable, and practical strategy for online EVs charging scheduling in corporate parking lots. It effectively manages real-world uncertainties while ensuring that available resources are optimally allocated.

The paper is organized as follows: in Section 2, a possible use case is described. The modeling of the smart parking system is detailed in Section 3, while the pricing scheme is discussed in Section 4. Sections 5 and 6 deal with the optimization problem and the related online management algorithm that tries to be consistent to a known offline management profile.

Extension to this is proposed in Section 7, where stochastic EVs behaviour together with deterministic ones is taken into account. This result can be exploited for real-time management but it is also a powerful way for computing the offline solution needed as data in the online algorithm of Section 6. The extended algorithm implementation related to the scenario approach is provided in Section 8. Finally, conclusions are summarized in Section 10.

Notation: $\mathbb{N}$ and ($\mathbb{R}^+$) $\mathbb{R}$ are the set of natural and (non-negative) real numbers, respectively; given a variable $\xi \in \mathbb{R}$, $\xi = \xi(t)$ indicates the value of ξ at the discrete time-step $t \in \mathbb{N}$. With some abuse of notation, $[t_1, t_2]$ with $t_1 \in \mathbb{N}$, $t_2 \in \mathbb{N}$, $t_1 \leq t_2$, indicates the set of consecutive integers from t_1 to t_2, while $[\xi_1, \xi_2]$ with $\xi_1 \in \mathbb{R}$, $\xi_2 \in \mathbb{R}$, $\xi_1 \leq \xi_2$, indicates the interval from ξ_1 to ξ_2. The star-superscript notation explicitly refer to an optimal value, while the barred notation indicates a provided given value (despite the fact that it will often be an optimal one in some sense). The notation $x_{i,\bullet}$ with $i, j \in \mathbb{N}$ indicates the value of the variable for the given index i and all the j.

2. Setup description and use case

The parking facility considered in this paper is owned and operated by a private company, that provides to the employee an area where recharge their EVs during the working day. For the sake of generality, it is assumed flexible working days, meaning employees can arrive and leave the job at different times during the day. Note that a fixed time can be seen as a particular case of the presented one.

Employees pay for charging their EVs. The corporate sets electricity buying and selling prices daily, purchasing energy to charge EVs and potentially selling surplus energy back to the grid. The company (that is the parking manager algorithm of the company) operates within known electricity price profiles throughout the day, both for purchasing and for selling energy. The parking management system operates across infra-day, day-ahead, and ancillary service markets, dynamically buying and selling energy to and from EVs during their parking time while also trading energy with the grid. Indeed, the aim of the company is to maximize the profit of the parking facility by leveraging energy market participation. By optimizing energy transactions and scheduling EVs efficiently, the company aims at maximizing its financial return, which could either increase company revenue or be redistributed among employees to incentivize EVs adoption in line with sustainability goals. The setting considers a possibly number of charging parking lots less than the total number of EVs owned by the corporate employees. Since charging stations are expensive, the company seeks to minimize the number of stations while ensuring high utilization. Therefore, rather than operating many underutilized stations for only a few hours daily, the company prioritizes a management strategy that schedules EVs for charging throughout the whole day.

In order to illustrate how the parking facility works, a typical usage scenario is described. Employees initially park their EVs in a general parking area (without charging stations) and move them to a designated charging station only during their assigned time slot. When an employee needs to recharge her EV, she notifies to the parking manager through an user interface (whose development goes beyond the scope of this work), specifying:

- Expected arrival time at work.
- Expected departure time.
- Desired SOC range at the end of the charging session (including the option for a full charge).
- Initial SOC upon arrival (either known or estimated by on board navigation algorithms that provides the arrival time and the expected consumption and/or residual battery charge).
- “Do not disturb" time slots, when she cannot put in charge or remove from charging her vehicle (e.g., due to meetings).

Charging slots can be booked in advance (e.g., up to a day ahead) or requested on short notice. This flexibility does not impact the proposed algorithm. Once the request is submitted, the parking manager assigns a charging station and a charging interval within the employee's working hours. The employee must move her vehicle from the general parking area to the assigned charging station at the designated time and remove it promptly at the end of the session. If no feasible charging schedule can accommodate the employee's request, the system notifies the unavailability. Additionally, the framework includes a reduced QoS option, where the system proposes an alternative charging schedule with a final SOC different from the originally requested level. The employee can accept or decline this offer.

Notice that it could happen that no solutions are possible for the data provided by the employee. In this case, the manager notifies the impossibility of the booking. The parking model is described in details in the next section.

3. Parking model

The considered parking facilities is made up of a set $\mathcal{R} = \{0, 2, \dots, R-1\}$, $R \in \mathbb{N}$, of charging stations, each uniquely identified by an integer. EVs requesting charging slots are represented by the set $\mathcal{V} \subset \mathbb{N}$, with each EV assigned an integer identifier. Given a finite time horizon for the allocation problem, EVs requesting charging outside this interval are assigned new identifiers, i.e., a single physical EV may possess multiple identifiers across different time intervals.

The assignment of i-th EV to charging station $j \in \mathcal{R}$ at the discrete time-step $t \in \mathbb{N}$ is modelled using the binary variable $a_{ij}(t) \in \{0, 1\}$, where $a_{ij}(t) = 1$ indicates an assignment and $a_{ij}(t) = 0$ otherwise. The set of EVs assigned at least once within the interval $[t_1, t_2]$, where $t_1 \leq t_2$, is defined as

$$\mathcal{A}(t_1, t_2) = \left\{ i \in \mathcal{V} : \exists t \in [t_1, t_2], \exists j \in \mathcal{R} \text{ s.t. } a_{ij}(t) = 1 \right\}.$$

Each i-th EV must specify its arrival time $\alpha_i \in \mathbb{N}$ and departure time $\delta_i \in \mathbb{N}$ (with $\alpha_i \leq \delta_i$) when requesting a charging session. The parking management algorithm operates within a bounded allocation interval $[S, F]$, with $S \in \mathbb{N}$, $F \in \mathbb{N}$, $S < F$, where F may represent the end of the workday or a defined horizon length.

Within the interval $[S, F]$, the set of EVs requesting charging is denoted by

$$Q(S, F) = \{ i \in \mathcal{V} : S \leq \alpha_i \leq F, S \leq \delta_i \leq F \}$$

This set is disjoint from the set of EVs already allocated, $\mathcal{A}(S, F)$, (that is $Q(S, F) \cap \mathcal{A}(S, F) = \emptyset$). Since an already assigned EV cannot request another assignment within the same interval. Both $Q(S, F)$ and $\mathcal{A}(S, F)$ are assumed to be finite.

The following subsections detail the constraints ensuring the feasibility of vehicle allocation and the optimization of energy consumption and resource allocation.

3.1. Vehicles charging power variables

Let us denote with $P^+_{ij}(t) \in \mathbb{R}^+$ ($P^-_{ij}(t) \in \mathbb{R}^+$) the power flowing from (to) the charging station j to (from) the vehicle i at time t. These power flows are constrained by the following limits

$$0 \leq P^+_{ij}(t) \leq z^{+,\max}_{i,\bullet}, \quad (1a)$$

$$0 \leq P^-_{ij}(t) \leq z^{-,\max}_{i,\bullet}, \quad (1b)$$

$\forall j \in \mathcal{R}, \forall i \in Q(S, F) \cup \mathcal{A}(S, F)$ and $t \in [S, F]$, where $z^{+,\max}_{i,\bullet}$ ($z^{-,\max}_{i,\bullet}$) indicates the maximum actual power flowing in (out) the i-th EV. Power exchange between vehicle i and station j at time t can only occur if the vehicle is assigned to the charging station, i.e., $a_{ij}(t) = 1$. To enforce this, we define the actual power entering and leaving vehicle i as follows

$$z^+_{ij}(t) = P^+_{ij}(t)a_{ij}(t) \text{ and } z^-_{ij}(t) = P^-_{ij}(t)a_{ij}(t).$$

These expressions ensure that no power is transferred when $a_{ij}(t) = 0$. Thus results in the following mixed integer linear constraints

$$\begin{cases} z^+_{ij}(t) \geq 0, \\ z^+_{ij}(t) \leq a_{ij}(t) z^{+,\max}_{i,\bullet}, \\ z^+_{ij}(t) \geq P^+_{ij}(t) - (1 - a_{ij}(t)) z^{+,\max}_{i,\bullet}, \\ z^+_{ij}(t) \leq P^+_{ij}(t), \end{cases} \tag{2}$$

and

$$\begin{cases} z^-_{ij}(t) \geq 0, \\ z^-_{ij}(t) \leq a_{ij}(t) z^{-,\max}_{i,\bullet}, \\ z^-_{ij}(t) \geq P^-_{ij}(t) - (1 - a_{ij}(t)) z^{-,\max}_{i,\bullet}, \\ z^-_{ij}(t) \leq P^-_{ij}(t), \end{cases} \tag{3}$$

$\forall j \in \mathcal{R}, \forall i \in \mathcal{Q}(S, F) \cup \mathcal{A}(S, F)$, and $t \in [S, F]$.

Each vehicle can be connected at most at one station that will be assigned by our proposed management algorithm as we will detail below. Therefore, being the assignment unknown at this stage, for convenience, we define the following variables

$$z^+_{i,\bullet}(t) = \sum_{j \in \mathcal{R}} z^+_{ij}(t), \tag{4}$$

and

$$z^-_{i,\bullet}(t) = \sum_{j \in \mathcal{R}} z^-_{ij}(t), \tag{5}$$

$\forall i \in \mathcal{Q}(S, F) \cup \mathcal{A}(S, F)$ and $t \in [S, F]$.

3.2. Vehicles power rate constraints

To protect battery life, the power entering and leaving each vehicle is subject to ramp rate constraints, limiting sudden changes in power flow. These constraints ensure smooth power variations over time

$$z^+_{i,\bullet}(t+1) - z^+_{i,\bullet}(t) \leq G^+_i, \tag{6a}$$

$$z^+_{i,\bullet}(t+1) - z^+_{i,\bullet}(t) \geq -G^+_i, \tag{6b}$$

$$z^-_{i,\bullet}(t+1) - z^-_{i,\bullet}(t) \leq G^-_i, \tag{6c}$$

$$z^-_{i,\bullet}(t+1) - z^-_{i,\bullet}(t) \geq -G^-_i, \tag{6d}$$

$\forall j \in \mathcal{R}, \forall i \in \mathcal{Q}(S, F) \cup \mathcal{A}(S, F)$ and $t \in [S, F-1]$, where G^+_i and G^-_i represent the ramp rates for vehicle i when receiving and supplying power, respectively. These constraints prevent excessive variations in charging and discharging power over consecutive time steps.

3.3. Vehicles assignment constraints

Each vehicle requesting a charging slot can be assigned to at most one charging station at any given time

$$\sum_{j \in \mathcal{R}} a_{ij}(t) \leq 1, \tag{7}$$

$\forall i \in \mathcal{Q}(S, F)$ and $t \in [S, F]$. This constraint ensures that an EV is never assigned to multiple charging stations simultaneously. Note that this condition applies only to vehicles requesting charging slots, i.e. EVs included in $\mathcal{Q}(S, F)$, and not to those already assigned, i.e. EVs included in $\mathcal{A}(S, F)$. The latter, in fact, must preserve their pre-scheduled allocations to avoid disrupting employee schedules. This is enforced by the following constraint

$$a_{ij}(t) = \bar{a}_{ij}(t), \tag{8}$$

$\forall i \in \mathcal{A}(S, F)$, $\forall j \in \mathcal{R}$ and $t \in [S, F]$, where $\bar{a}_{ij}(t)$ represents the previously determined assignment status. Thus assures that assigned vehicles maintain their scheduled slots, preventing any changes that could disrupt previously planned allocations.

To prevent scenarios where a vehicle temporarily leaves a charging slot and later re-parks, the assigned charging period must consist of consecutive time slots within the interval $[S, F]$. This is enforced through the following constraints

$$a_{ij}(t) - a_{ij}(t-1) + 1 \geq a_{ij}(h), \;\; h = t+1, \ldots, F, \tag{9a}$$

$$a_{ij}(t) - a_{ij}(t-1) + 1 \geq a_{ik}(p), \;\; p = t, \ldots, F, \;\; k \in \mathcal{R} - \{j\}, \tag{9b}$$

$\forall j \in \mathcal{R}, \forall i \in \mathcal{Q}(S, F)$ and $t \in [S+1, F]$. Each vehicle can only be assigned a charging slot within its arrival and departure window

$$a_{ij}(t) = 0, \tag{10}$$

$\forall i \in \mathcal{Q}(S, F)$ and $t \in [S, \alpha_i - 1] \cup [\delta_i + 1, F]$. Thus guarantees that no vehicle is scheduled for charging before its arrival time α_i or after its departure time δ_i.

Employees may specify *unavailability periods* during which they cannot move their vehicle (e.g., due to meetings). For each vehicle i let $\mathcal{D}_i \subset [\alpha_i, \delta_i]$, denote the set of unavailable time slots. The following constraint ensures that charging assignments remain unchanged during these intervals

$$a_{ij}(t) = a_{ij}(t-1), \tag{11}$$

$\forall i \in \mathcal{Q}(S, F)$ and $t \in \mathcal{D}_i$, where the initial charging status is set as $a_{ij}(\alpha_i - 1) = 0$. This prevents the system from scheduling charging operations at times when the vehicle cannot be moved.

Constraints (6)–(11) collectively define a feasible and structured charging schedule, balancing efficient resource allocation, user-defined constraints, and grid stability while ensuring compliance with operational requirements

3.4. SOC constraints

The battery model includes the charging and discharging dynamics as in [27]. The evolution of the SOC $e_i \in [0, 1] \subset \mathbb{R}^+$ for the i-th vehicle is expressed as

$$e_i(t+1) = \lambda_i e_i(t) + \tau \left[\eta^+_i z^+_{i,\bullet}(t) - \frac{1}{\eta^-_i} z^-_{i,\bullet}(t) \right], \tag{12}$$

$\forall i \in \mathcal{A}(S, F) \cup \mathcal{Q}(S, F)$ and $t \in [\alpha_i, \delta_i]$, where τ is the sampling period, $e_i(\alpha_i) = e_{i,\alpha_i}$ represents the initial battery SOC upon arrival, $\lambda_i \in (0, 1] \subset \mathbb{R}^+$ accounts for the self-discharging losses and $\eta_i^+, \eta_i^- \in (0, 1] \subset \mathbb{R}^+$ are the charging/discharging efficiencies.

To ensure feasibility, the SOC always remains within valid operational limits

$$e_i(t) \geq e_i^{\min}, \quad e_i(t) \leq e_i^{\max}, \tag{13}$$

with $e_i^{\min} < e_i^{\max}$, $\forall i \in \mathcal{A}(S,F) \cup Q(S,F)$ and $t \in [\alpha_i, \delta_i]$. At departure, the vehicle SOC should ideally fall within the desired SOC interval specified by the owner, denoted as $[e^-_{i,\delta_i}, e^+_{i,\delta_i}]$. Nevertheless, to increase scheduling flexibility, the parking manager may propose a reduced QoS by allowing some relaxation of these SOC bounds. The following constraints codifies this feature

$$e_i(\delta_i) \leq e^+_{i,\delta_i} + y^+_{\delta,i}, \tag{14a}$$
$$e_i(\delta_i) \geq e^-_{i,\delta_i} - y^-_{\delta,i}, \tag{14b}$$
$$y^+_{\delta,i} \geq 0, \tag{14c}$$
$$y^-_{\delta,i} \geq 0, \tag{14d}$$
$$y^+_{\delta,i} \leq y^{+,\max}_{\delta,i}, \tag{14e}$$
$$y^-_{\delta,i} \leq y^{-,\max}_{\delta,i}, \tag{14f}$$

$\forall i \in \mathcal{A}(S,F) \cup Q(S,F)$, where the non-negative variables $y^-_{\delta,i}$, $y^+_{\delta,i}$ represent potential deviations from the requested SOC range and the constants $y^{+,\max}_{\delta,i}$, $y^{-,\max}_{\delta,i}$ indicate the maximum allowed QoS flexibility. Setting $y^{+,\max}_{\delta,i} = y^{-,\max}_{\delta,i} = 0$ ensures that the final SOC must exactly match the requested interval. In contrast, setting both values to a sufficiently large constant (e.g., greater than $e_i^{\max}$) allows total relaxation, expanding the solution space. This enables a trade-off between feasibility and user satisfaction, where the system can suggest minor deviations to improve overall scheduling efficiency while keeping the employee informed of any SOC reductions.

3.5. Charging station constraints

In the general setting we consider, a charging station j can serve up to N_j vehicles simultaneously. This is encoded as

$$\sum_{i \in \mathcal{A}(S,F) \cup Q(S,F)} a_{ij}(t) \leq N_j, \tag{15}$$

$\forall j \in \mathcal{R}$ and $t \in [S,F]$. Notice that while some advanced charging station support multiple concurrent connections, most existing infrastructures can only charge a vehicle at the time. This is obviously represented by operating with $N_j = 1$. Beside constraint (15), each charging station has a maximum power that can be delivered (taken) to (from) the vehicles, denoted as $z^{+,\max}_{\bullet,j} \in \mathbb{R}^+$ ($z^{-,\max}_{\bullet,j} \in \mathbb{R}^+$), respectively. The total power supplied or extracted at any given time, must satisfy the following constraints

$$\sum_{i \in \mathcal{A}(S,F) \cup Q(S,F)} z^+_{ij}(t) \leq z^{+,\max}_{\bullet,j}, \quad \sum_{i \in \mathcal{A}(S,F) \cup Q(S,F)} z^-_{ij}(t) \leq z^{-,\max}_{\bullet,j}, \tag{16}$$

$\forall j \in \mathcal{R}$ and $t \in [S,F]$.

Certain charging stations may be unavailable at specific times due to maintenance or other operational constraints. Let $\mathcal{U}_j(S,F) \subseteq [S,F]$ represent the unavailable time slots for station j. The following constraint ensures that no vehicle is assigned to an unavailable charging station

$$a_{ij}(t) = 0, \tag{17}$$

$\forall j \in \mathcal{R}$, $\forall i \in \mathcal{A}(S,F) \cup Q(S,F)$ and $t \in \mathcal{U}_j(S,F)$.

3.6. Power grid connection

The parking manager participates in the infra-day market (idm), day-ahead market (dam) and the ancillary service market (asm). The manager can either buy or sell energy on these markets [27]. Therefore we consider the powers bought ($P^+_{\text{idm}}(t)$, $P^-_{\text{idm}}(t)$, $P^+_{\text{dam}}(t)$) and sold ($P^-_{\text{dam}}(t)$, $P^+_{\text{asm}}(t)$, $P^-_{\text{asm}}(t)$) on these markets at time t. Within the time interval $[S,F]$, the following constraints are given for the idm and dam:

$$P^+_{\text{idm}}(t) \geq 0, \tag{18a}$$
$$P^-_{\text{idm}}(t) \geq 0, \tag{18b}$$
$$P^+_{\text{dam}}(t) \geq 0, \tag{18c}$$
$$P^-_{\text{dam}}(t) \geq 0. \tag{18d}$$

The asm differs from the other markets as it involves real-time power adjustments in response to grid needs. To do so, the parking manager offers power flexibility through upward (selling power) and downward (buying power) adjustments. Therefore, within the time interval $[S,F]$, we have the following constraint

$$s^+(t) \geq 0, \tag{19a}$$
$$s^+(t) \leq s^{+,\max}, \tag{19b}$$
$$s^-(t) \geq 0, \tag{19c}$$
$$s^-(t) \leq s^{-,\max}, \tag{19d}$$

where $s(t)^-$ and $s(t)^+$ are the upward and downward adjustment and $s(t)^{-,\max} \in \mathbb{R}^+$ and $s(t)^{+,\max} \in \mathbb{R}^+$ the relative maximum values. The actual participation to the ancillary market by the parking manager at time t is denoted with the boolean variable $m(t) \in \{0,1\}$, where true value stands for participation, false otherwise. Therefore, a power flexibility variation ($z^-_{\text{asm}}(t)$, $z^+_{\text{asm}}(t)$) can only be offered when the (time) corresponding boolean participation variable is true, that is $z^-_{\text{asm}}(t) = s^-(t)m(t)$ and $z^+_{\text{asm}}(t) = s^+(t)m(t)$. These are rewritten as the following mixed integer linear constraints

$$\begin{cases} z^+_{\text{asm}}(t) \geq 0, \\ z^+_{\text{asm}}(t) \leq m(t)s^{+,\max}, \\ z^+_{\text{asm}}(t) \geq s^+(t) - (1 - m(t))s^{+,\max}, \\ z^+_{\text{asm}}(t) \leq s^+(t), \end{cases} \tag{20}$$

and

$$\begin{cases} z^-_{\text{asm}}(t) \geq 0, \\ z^-_{\text{asm}}(t) \leq m(t)s^{-,\max}, \\ z^-_{\text{asm}}(t) \geq s^-(t) - (1 - m(t))s^{-,\max}, \\ z^-_{\text{asm}}(t) \leq s^-(t), \end{cases} \tag{21}$$

Following [27], the downward (power buying) and upward (power selling) ancillary service participation is

$$P^{+}_{\text{asm}}(t) = z^{+}_{\text{asm}}(t)\omega^{+}(t), \quad P^{-}_{\text{asm}}(t) = z^{-}_{\text{asm}}(t)\omega^{-}(t), \tag{22}$$

where $\omega^{-}(t), \omega^{+}(t) \in [0, 1] \subset \mathbb{R}^{+}$ are the downward and upward service signals provided by the transmission system operator (TSO). Since the TSO determines $\omega^{+}(t)$ and $\omega^{-}(t)$ in real-time, these values are unknown when the parking manager submits its ancillary service profile.

In some countries, the ancillary service market participation further requires a minimum size of consecutive time instants, namely T_{asm} at which the flexibility must be guaranteed. In our model, this constraint is given by

$$m(t) - m(t-1) \leq m(h), \tag{23}$$

with $h = t+1, \ldots, \min\{t + T_{\text{asm}} - 1, F\}$, $\forall t \in [S+1, F]$.

In view of the fact that the parking facility is connected to a single grid dispatch point, the total power balance, within the time interval $[S, F]$ must be satisfied at each time instant

$$\sum_{i \in \mathcal{A}(S,F) \cup \mathcal{Q}(S,F)} \left[z^{+}_{i,\bullet}(t) - z^{-}_{i,\bullet}(t) \right] - P^{+}_{\text{idm}}(t) + P^{-}_{\text{idm}}(t)$$
$$- P^{+}_{\text{dam}}(t) + P^{-}_{\text{dam}}(t) - P^{+}_{\text{asm}}(t) + P^{-}_{\text{asm}}(t) = 0. \tag{24a}$$
$$P^{+}_{\text{idm}}(t) + P^{+}_{\text{dam}}(t) + P^{+}_{\text{asm}}(t) \leq P^{+,\max}, \tag{24b}$$
$$P^{-}_{\text{idm}}(t) + P^{-}_{\text{dam}}(t) + P^{-}_{\text{asm}}(t) \leq P^{-,\max}. \tag{24c}$$

Since participation in the day-ahead and ancillary service markets must be pre-decided before online scheduling, the following values are fixed during real-time decision-making within the time interval $[S, F]$

$$P^{+}_{\text{dam}}(t) = \bar{P}^{+}_{\text{dam}}(t), \tag{25a}$$
$$P^{-}_{\text{dam}}(t) = \bar{P}^{-}_{\text{dam}}(t), \tag{25b}$$
$$s^{+}(t) = \bar{s}^{+}(t), \tag{25c}$$
$$s^{-}(t) = \bar{s}^{-}(t), \tag{25d}$$
$$m(t) = \bar{m}(t). \tag{25e}$$

Remark 1. The constraints provided in this section do not explicitly prevent power from simultaneously flowing into and out of the grid. Likewise (12) does not inherently exclude scenarios where a vehicle is both charging and discharging at the same time. However, such cases are physically meaningless and must be avoided. A straightforward approach to prevent these unrealistic situations would be to introduce mutual exclusivity constraints for power inflow and outflow. However, the structure of the optimization problem itself excludes these solutions, eliminating the need for additional constraints. A detailed explanation is provided in the appendix.

4. Costs

The pricing model follows [27], with the extension to the infra-day market. At each time instant t, the *unit energy price* paid by an EV to the parking manager for charging is denoted as $c^{+}_{v}(t) \in \mathbb{R}^{+}$. Conversely, when the parking manager buys energy from an EV (i.e., vehicle-to-grid discharging), the unit price paid to the EV is $c^{-}_{v}(t) \in \mathbb{R}^{+}$. The unit prices for energy purchased from the grid through different markets are indicated with the real positive variables $c^{+}_{\text{idm}}(t)$, $c^{+}_{\text{dam}}(t)$ and $c^{+}_{\text{asm}}(t)$ respectively, while, the prices at which the parking manager sells energy back to the grid are $c^{-}_{\text{idm}}(t)$, $c^{-}_{\text{dam}}(t)$ and $c^{-}_{\text{asm}}(t)$. For a realistic and meaningful energy market these prices must satisfy the following relationships

$$c^{-}_{\text{idm}}(t) < c^{-}_{\text{dam}}(t) < c^{+}_{\text{dam}}(t) < c^{+}_{\text{idm}}(t) < c^{+}_{v}(t) < c^{-}_{v}(t),$$
$$c^{-}_{\text{dam}}(t) < c^{-}_{\text{asm}}(t) < c^{-}_{v}(t),$$
$$c^{+}_{\text{dam}}(t) < c^{+}_{\text{asm}}(t) < c^{+}_{\text{v}}(t).$$

To ensure that borrowing energy from a vehicle and returning it later does not result in a financial loss for the EV owner, the pricing scheme must satisfy the constraint

$$c^{-}_{v}(t) > \max_{i \in \mathcal{V}, t' \in [S,F]} \frac{1}{\eta^{+}_{i}\eta^{-}_{i}} c^{+}_{v}(t'). \tag{26}$$

These energy prices contribute to the objective function of the optimization problem formulated below.

4.1. Vehicles recharging cost function

The total cost of energy transactions at each time t includes multiple components, accounting for EVs charging discharging, market participation, and QoS reductions. The cost (or revenue, if negative) associated with charging and discharging EVs at time t is

$$C_{v}(t) = c^{-}_{v}(t)\tau \sum_{i \in \mathcal{A}(S,F) \cup \mathcal{Q}(S,F)} z^{-}_{i,\bullet}(t) - c^{+}_{v}(t)\tau \sum_{i \in \mathcal{A}(S,F) \cup \mathcal{Q}(S,F)} z^{+}_{i,\bullet}(t). \tag{27}$$

where τ is the sampling period. This expression captures the revenue from EV discharging ($z^{-}_{i,\bullet}(t)$), i.e., energy sold by vehicle to the parking manager at price $c^{-}_{v}(t)$ and the cost of EV charging ($z^{+}_{i,\bullet}(t)$), i.e., energy supplied by the parking manager to vehicle at price $c^{+}_{v}(t)$.

4.2. Market participation cost functions

The cost (or revenue) for participating in the infra-day market per unit of time is

$$C_{\text{idm}}(t) = c^{+}_{\text{idm}}(t)\tau P^{+}_{\text{idm}}(t) - c^{-}_{\text{idm}}(t)\tau P^{-}_{\text{idm}}(t). \tag{28}$$

The cost (or revenue) for participating to the day-ahead market per unit of time is

$$C_{\text{dam}}(t) = c^{+}_{\text{dam}}(t)\tau P^{+}_{\text{dam}}(t) - c^{-}_{\text{dam}}(t)\tau P^{-}_{\text{dam}}(t). \tag{29}$$

Analogously to the case of the infra-day and day-ahead, the cost (or revenue) from participation in the ancillary service market is

$$C_{\text{asm}}(t) = c^{+}_{\text{asm}}(t)\tau P^{+}_{\text{asm}}(t) - c^{-}_{\text{asm}}(t)\tau P^{-}_{\text{asm}}(t). \tag{30}$$

4.3. QoS reduction cost function

To discourage excessive deviations from the originally requested SOC levels, a penalty cost is introduced when the parking manager applies QoS reductions. The associated cost is given by

$$C_y = \sum_{i\in Q(S,F)} w^+_{y,i} y^+_{\delta,i} + \sum_{i\in Q(S,F)} w^-_{y,i} y^-_{\delta,i}, \tag{31}$$

where $y^+_{\delta,i}$ represents the increase in the final SOC beyond the originally requested maximum, $y^-_{\delta,i}$ represents the decrease in the final SOC below the originally requested minimum, and $w^+_{y,i}$ and $w^-_{y,i}$ are the corresponding penalty coefficients applied to these deviations. By incorporating this penalty, the optimization framework encourages adherence to the original SOC requests while allowing limited flexibility when necessary.

4.4. Overall power cost function

The constraints described above are essential for computing the total power cost, which is used in one of the two optimization problems formulated later in this work. The overall allocation cost for a given scheduling interval $[S, F]$ is given by

$$J_c = \sum_{t=S}^{F} [C_v(t) + C_{\mathrm{idm}}(t) + C_{\mathrm{dam}}(t) + C_{\mathrm{asm}}(t)] + C_y. \tag{32}$$

This cost function accounts for the energy transactions related to EVs charging discharging, participation in the infra-day, day-ahead, and ancillary service markets, and any penalties associated with QoS reductions.

4.5. Deviation from the offline scenario cost function

In this part of the manuscript it is assumed that a day-ahead solution has already been pre-computed over historic/forecasted data. An approach for solving this offline problem is described in the next following sections. Here, we focus on how the real-time (run-time) problem should incorporate these offline results.

The runtime scheduling algorithm aims to adhere as closely as possible to the offline solution while accommodating discrepancies between offline precomputed data, based on forecasts, and real-time data, based on actual conditions. To enforce this adherence we introduce a deviation cost function

$$J_a = \sum_{t=S}^{F} \sum_{i\in Q(S,F)} \sum_{j\in\mathcal{R}} w_{ij}(t) a_{ij}(t). \tag{33}$$

The weight coefficients $w_{ij}(t)$ are designed to penalize deviations from the offline solution, meaning that allocation $a_{ij}(t)$ that differs from precomputed assignments incurs higher costs. While the specific method for selecting $w_{ij}(t)$ does not affect the optimization framework, a simple yet effective choice is considering $w_{ij}(t) = -1, \forall j \in \mathcal{R}$ if vehicle i is allocated at time t (irrespective of which specific charging station) and $w_{ij}(t) = 0, \forall j \in \mathcal{R}$ otherwise. This selection encourages the real-time algorithm to maintain the same time slots of the offline solution while allowing flexibility in choosing different charging stations if needed. This approach is adopted for the numerical validation presented later in the paper.

5. Optimal solution for the parking management problem

To solve the runtime problem, we adopt a two-stages sequential optimization. First (stage 1) the optimization problem aims to align the real-time allocation as closely as possible to the precomputed offline schedule, considering both previously assigned vehicles and new charging requests. Second (stage 2) a further optimization problem minimizes the total power cost given the allocation obtained in stage 1

5.1. Stage 1: optimal deviation from the offline solution

The first optimization problem, assigns new charging slots for the set of incoming vehicles $Q(S, F)$, while ensuring the previously allocated vehicles $\mathcal{A}(S, F)$ retain their pre-scheduled charging sessions. The objective is to minimize the deviation cost (33) by solving the following problem

$$\begin{aligned} &\min \; J_a \\ &\text{s.t.} \\ &\quad \text{Vehicles charging power variables (1)} - (5), \\ &\quad \text{Vehicles power ramp rate constraints (6)}, \\ &\quad \text{Vehicles assignment constraints (7)} - (8), \\ &\quad \text{SOC constraints (12)} - (14), \\ &\quad \text{chargingstation constraints (15)} - (17), \\ &\quad \text{Power grid connection (18)} - (25). \end{aligned} \tag{34}$$

Notice that after having solved problem (34) each vehicle i is either successfully assigned a charging slot or remains unassigned if no feasible solution exists. When assigned, the vehicle is allocated to a unique interval $[\bar{\alpha}_i, \bar{\delta}_i] \subseteq [\alpha_i, \delta_i]$ consisting of consecutive time slots, ensuring continuity in the charging process.

5.2. Stage 2: optimal power solution

Let us suppose (34) is solved and J^*_a is the obtained optimal cost by taking into account the sets $Q(S, F)$ and $\mathcal{A}(S, F)$. Then, considering the same sets $Q(S, F)$ and $\mathcal{A}(S, F)$, the following optimization problem is designed to exploit potential degree of freedom in the allocation feasibility space to minimize the overall power cost.

$$\begin{aligned} &\min \; J_c \\ &\text{s.t.} \\ &\quad (1) - (25), \\ &\quad J_a \le J^*_a. \end{aligned} \tag{35}$$

This ensures that while minimizing power costs, the optimization does not introduce excessive deviations from the offline schedule computed in stage 1.

Key observations include *online data compensation*, where unlike the offline problem that relies on forecasts, both (34) and (35) operate on real-time data, naturally correcting deviations between predicted and actual conditions to improve allocation accuracy. Additionally, *flexibility in cost function design* allows problem (35) to be further extended with additional objectives, such as charging station load balancing to ensure uniform ageing of equipment. This two-stage approach effectively balances adherence to the offline schedule with cost efficiency, providing a practical, online solution for managing EV charging in a dynamic corporate environment.

6. Runtime management algorithm

The optimization problems (34) and (35) are encapsulated in a runtime algorithm detailed in what follows. All the requests are collected in a FIFO queue, say it C. When a customer requests for a recharging time slots, she provides its arrival and departure times $[\alpha_i, \delta_i]$, possible unavailability periods $\mathcal{D}_i$, the initial and the final range of SOC, i.e. e_{i,α_i} and $[e^-_{i,\delta_i}, e^+_{i,\delta_i}]$, and the possible allowed flexibility, $y^{-,\max}_{\delta,i}$ and $y^{+,\max}_{\delta,i}$.

We consider the hypothesis that different requests are generated at different time instants. Excluding multiple requests at the same time is motivated by the fact that the whole parking management systems is developed as a cloud service, where the time each request is inserted into the queue C after being received is so fast to make the possibility of having two or more booking events at the same time practically impossible. However, in the rare case of simultaneous requests, a random ordering can be assigned.

The algorithm is executed any time a triggering event is detected. Thus corresponds to the fact that queue C is not empty, i.e., at least one EV is requesting a recharge. An high-level description of the process involved is reported in Algorithm 1. Notice that, to the sake of clarity, we have omitted the data needed to the optimization problems (34) and (35). Moreover we report as output only the arrival and the departure time of each i-th EV. Indeed the booking is notified to the customer only with park and pick up times. Power profiles are instead used "internally" to provide the references, that each charging station has to follow. These reference values may be updated dynamically each time Algorithm 1 is executed, but such adjustments occur transparently and do not affect the EV owner's experience.

Algorithm 1: Process of the parking manager

```
Input: C, S ∈ ℝ, F ∈ ℝ
Output: 𝒜(S,F), Q(S,F), ᾱ_i, δ̄_i
Q(S,F) ← ∅;
for i ∈ C do
    Q(S,F) ← Q(S,F) ∪ {i};
    C ← C\{i};
    //solve the optimization problem ;
    Solve (34);
    if (34) is feasible then
        Solve (35) which provides [ᾱ_i, δ̄_i];
        if EV i accepts [ᾱ_i, δ̄_i] then
            𝒜(S,F) ← 𝒜(S,F) ∪ {i};
        end
    else
        Notify to the EV i the allocation fail;
    end
end
```

7. Scenario approach

The optimization problems proposed in Section 5.1 and Section 5.2 are instrumental for the real time management in Algorithm 1.

In this section we change the viewpoint and focus not only on deterministic aspects but also on optimizing the EVs allocation according to statistical data. Specifically, together with the deterministic set $Q(S,F)$ of vehicles requesting allocation and those already allocated (set $\mathcal{A}(S,F)$), we introduce an additional finite set of vehicles, denoted as $Q'(S,F)$. This set represents vehicles that are expected to request charging within the time interval $[S,F]$ but have not yet submitted a request at the time the optimization problem is executed. Set $Q'(S,F)$ models historically forecasted employees that are expected to ask to recharge their EVs. The three sets are mutually exclusive, ensuring that with $Q(S,F) \cap Q'(S,F) = \emptyset$ and $\mathcal{A}(S,F) \cap Q'(S,F) = \emptyset$.

Since vehicles in $Q'(S,F)$ have not formally requested charging, their characteristics, such as arrival and departure times α_i and δ_i, unavailability periods $\mathcal{D}_i$, initial SOC $e_i(\alpha_i)$, and desired SOC range $[e^-_{i,\delta_i}, e^+_{i,\delta_i}]$, are modelled as independent stochastic variables, each characterized by a known probability density function. Because the initial SOC is stochastic, the battery dynamics in (12) also become a stochastic process, meaning that $e_i(t)$ itself is a stochastic variable.

To differentiate between different realizations of these stochastic variables, we use scenario indexing. Specifically, for each vehicle $i \in Q'(S,F)$, we denote by i^k the k-th realization of vehicle i, meaning that α_{ik} represents the arrival time of i-th EV in the k-th scenario. The same notation applies to all other stochastic variables.

Finally, being the downward and upward service signals $\omega^-(t)$ and $\omega^+(t)$ unknown at the time the optimization problem described later is run, here they are treated as stochastic variables too.

To manage the parking in such stochastic setting, and to compute the decision variables $s^-(t), s^+(t)$ that we considered given in problems (35), (34) (constraints (25)), we adopt a scenario based approach with sample average approximation. A full description of these methods is beyond the scope of this paper, for theoretical details refer to [28] and to [29, 30] for applications in energy management problems.

In short, the scenario based approach consists in generating a sufficiently high number $K \in \mathbb{N}$ of variables realizations. Each variables realization is called *scenario* and gives rise to a deterministic set of constraints. The optimization is conducted considering all the scenarios simultaneously and minimizing the overall constraints violation.

In view of this, modifications to some of the constraints derived in Section 3 and costs in Section 4 are needed. The latter are provided in the following subsections.

7.1. Modified constraints for the scenario setting

The modifications to constraints (1)–(9) and (15)–(17) account for the inclusion of both deterministic requests and the additional stochastic set, i.e. $Q(S,F) \cup Q'(S,F)$. The resulting constraints are indicated in the following as (1)′–(9)′ and (15)′–(17)′. Other constraints need to be properly designed to account for the stochastic setting. They are designed in what follows.

For the EVs in $\mathcal{Q}'(S,F)$, constraints (10) are removed. Given the realization i^k, we define an additional cost function to minimize allocation deviations

$$Q^a_{i^k} = \sum_{j\in\mathcal{R}} \sum_{t=S}^{F} q^a_{i^k}(t) a_{ij}(t),$$

where the non negative weights $q^a_{i^k}(t)$ are defined such that $q^a_{i^k}(t) = 0$, $t \in [S, \alpha_{i^k} - 1] \cup [\delta_{i^k} + 1, F]$. A suitable choice for $q^a_{i^k}(t)$ is to assign increasing values as t is far from the interval $[\alpha_{i^k}, \delta_{i^k}]$, e.g. $q^a_{i^k}(t) = \alpha_{i^k} - t$, for $t \le \alpha_{i^k}$ and $q^a_{i^k}(t) = t - \delta_{i^k}$, for $t \ge \delta_{i^k}$. From this, we define the average allocation cost across scenarios as

$$Q^a_i = \frac{1}{K}\sum_{k=1}^{K} Q^a_{i^k},$$

and the total allocation cost over all stochastic vehicles

$$Q^a = \sum_{i\in\mathcal{Q}'(S,F)} Q^a_i. \tag{36}$$

Constraint (11) result in

$$a_{ij}(t) - a_{ij}(t-1) \le \zeta^{\mathcal{D},+}_{i^k,j}(t), \tag{37a}$$
$$a_{ij}(t-1) - a_{ij}(t) \le \zeta^{\mathcal{D},-}_{i^k,j}(t), \tag{37b}$$
$$\zeta^{\mathcal{D},+}_{i^k,j}(t) \ge 0, \tag{37c}$$
$$\zeta^{\mathcal{D},-}_{i^k,j}(t) \ge 0, \tag{37d}$$

$\forall j \in \mathcal{R}$ and $t \in [S+1, F]$ with $\zeta^{\mathcal{D},-}_{i^k,j}(t)$ and $\zeta^{\mathcal{D},+}_{i^k,j}(t)$ additional decision variables. This results in the deviation penalty cost

$$Q^{\mathcal{D}}_{i^k} = \sum_{j\in\mathcal{R}} \sum_{t=S+1}^{F} q^{\mathcal{D}}_{i^k}(t)(\zeta^{\mathcal{D},-}_{i^k,j}(t) + \zeta^{\mathcal{D},+}_{i^k,j}(t)),$$

where $q^{\mathcal{D}}_{i^k}(t) = 0$ for $t \notin \mathcal{D}_{i^k}$ and strictly positive otherwise. The scenario-based cost function is then defined as

$$Q^{\mathcal{D}}_i = \frac{1}{K}\sum_{k=1}^{K} Q^{\mathcal{D}}_{i^k},$$

and

$$Q^{\mathcal{D}} = \sum_{i\in\mathcal{Q}'(S,F)} Q^{\mathcal{D}}_i. \tag{38}$$

Battery dynamics are modified, for those $i \in \mathcal{Q}'(S,F)$, as

$$e_{i^k}(t+1) = \lambda_i e_{i^k}(t) + \tau\left[\eta^+_i z^+_{i,\bullet}(t) - \frac{1}{\eta^-_i} z^-_{i,\bullet}(t)\right], \tag{39}$$

for $t \in [S, F-1]$ with $e_{i^k}(\alpha_{i^k}) = e_{i^k,\alpha_{i^k}}$ initial condition.

The related constraints assume the form

$$e_{i^k}(t) \ge e^{\min}_i - \zeta^{e,-}_{i^k}(t), \tag{40a}$$
$$e_{i^k}(t) \le e^{\max}_i + \zeta^{e,+}_{i^k}(t), \tag{40b}$$
$$\zeta^{e,+}_{i^k}(t) \ge 0, \tag{40c}$$
$$\zeta^{e,-}_{i^k}(t) \ge 0, \tag{40d}$$

$\forall t \in [S, F]$ and

$$e_{i^k}(\alpha_{i^k}) \le e_{i^k,\alpha_{i^k}} + y^+_{\alpha,i^k}, \tag{41a}$$
$$e_{i^k}(\alpha_{i^k}) \ge e_{i^k,\alpha_{i^k}} - y^-_{\alpha,i^k}, \tag{41b}$$
$$y^+_{\alpha,i^k} \ge 0, \tag{41c}$$
$$y^-_{\alpha,i^k} \ge 0. \tag{41d}$$
$$e_{i^k}(\delta_{i^k}) \le e^+_{i^k,\delta_{i^k}} + y^+_{\delta,i^k}, \tag{41e}$$
$$e_{i^k}(\delta_{i^k}) \ge e^-_{i^k,\delta_{i^k}} - y^-_{\delta,i^k}, \tag{41f}$$
$$y^+_{\delta,i^k} \ge 0, \tag{41g}$$
$$y^-_{\delta,i^k} \ge 0. \tag{41h}$$

Constraints in (40) are exploited to penalize the SOC deviations from the given limits. To do so, the following cost functions are introduced

$$Q^e_{i^k} = \sum_{t=S}^{F} q^e_i(\zeta^{e,-}_{i^k}(t) + \zeta^{e,+}_{i^k}(t)),$$

with q^e_i positive weight that does not depend on the particular realization (conversely to (36) and (38)) and

$$Q^e_i = \frac{1}{K}\sum_{k=1}^{K} Q^e_{i^k}, \quad Q^e = \sum_{i\in\mathcal{Q}'(S,F)} Q^e_i. \tag{42}$$

Constraints in (41) are instead used to penalize the violation on the initial and desired final SOC. To do so, the following cost are defined

$$Q^\alpha_i = \frac{1}{K}\sum_{k=1}^{K} q^\alpha_i(y^-_{\alpha,i^k} + y^+_{\alpha,i^k}), \quad Q^\delta_i = \frac{1}{K}\sum_{k=1}^{K} q^\delta_i(y^-_{\delta,i^k} + y^+_{\delta,i^k}),$$

with q^α_i and q^δ_i positive weights and

$$Q^\alpha = \sum_{i\in\mathcal{Q}'(S,F)} Q^\alpha_i, \quad Q^\delta = \sum_{i\in\mathcal{Q}'(S,F)} Q^\delta_i. \tag{43}$$

When considering the power exchanged with the grid, since the TSO service signals are not known and, therefore, modeled in a stochastic way, equations (22) are replaced by

$$P^+_{\mathrm{asm},k}(t) = z^+_{\mathrm{asm}}(t)\omega^+_k(t), \tag{44a}$$
$$P^-_{\mathrm{asm},k}(t) = z^-_{\mathrm{asm}}(t)\omega^-_k(t), \tag{44b}$$

where the subscript k denotes the k-th realization scenario. Consequently, constraint (24) is replaced by the following relaxed one

$$\sum_{i\in\mathcal{A}(S,F)\cup\mathcal{Q}(S,F)\cup\mathcal{Q}'(S,F)} \left[z^+_{i,\bullet}(t) - z^-_{i,\bullet}(t)\right] - P^+_{\rm idm}(t) - P^+_{\rm idm}(t) + P^-_{\rm dam}(t) + P^-_{\rm dam}(t) - P^+_{{\rm asm},k}(t) + P^-_{{\rm asm},k}(t) \le \zeta_k^{{\rm bal},+}(t), \quad (45a)$$

$$\sum_{i\in\mathcal{A}(S,F)\cup\mathcal{Q}(S,F)\cup\mathcal{Q}'(S,F)} \left[z^+_{i,\bullet}(t) - z^-_{i,\bullet}(t)\right] - P^+_{\rm idm}(t) - P^+_{\rm idm}(t) + P^-_{\rm dam}(t) + P^-_{\rm dam}(t) - P^+_{{\rm asm},k}(t) + P^-_{{\rm asm},k}(t) \ge -\zeta_k^{{\rm bal},-}(t), \quad (45b)$$

$$P^+_{\rm idm}(t) + P^+_{\rm dam}(t) + P^+_{{\rm asm},k}(t) \le P^{+,\max} + \zeta_k^{{\rm buy},+}(t) \quad (45c)$$

$$P^+_{\rm idm}(t) + P^+_{\rm dam}(t) + P^+_{{\rm asm},k}(t) \ge -\zeta_k^{{\rm buy},-}(t) \quad (45d)$$

$$P^-_{\rm idm}(t) + P^-_{\rm dam}(t) + P^-_{{\rm asm},k}(t) \le P^{-,\max} + \zeta_k^{{\rm sell},+}(t) \quad (45e)$$

$$P^-_{\rm idm}(t) + P^-_{\rm dam}(t) + P^-_{{\rm asm},k}(t) \ge -\zeta_k^{{\rm sell},-}(t) \quad (45f)$$

$$\zeta_k^{{\rm bal},+}(t) \ge 0, \quad (45g)$$

$$\zeta_k^{{\rm bal},-}(t) \ge 0, \quad (45h)$$

$$\zeta_k^{{\rm buy},+}(t) \ge 0, \quad (45i)$$

$$\zeta_k^{{\rm buy},-}(t) \ge 0, \quad (45j)$$

$$\zeta_k^{{\rm sell},+}(t) \ge 0, \quad (45k)$$

$$\zeta_k^{{\rm sell},-}(t) \ge 0, \quad (45l)$$

$\forall t \in [S, F]$. To the above constraints we associate the following cost function so as to minimize power imbalance on the scenarios

$$Q_k^P = q^P \sum_{t=S}^{F} \Big(\zeta_k^{{\rm bal},-}(t) + \zeta_k^{{\rm bal},+}(t) + \zeta_k^{{\rm buy},-}(t) + \zeta_k^{{\rm buy},+}(t) + \zeta_k^{{\rm sell},-}(t) + \zeta_k^{{\rm sell},+}(t)\Big),$$

with q^P positive weight and

$$Q^P = \frac{1}{K}\sum_{k=1}^{K} Q_k^P. \quad (46)$$

7.2. *Modified costs for the scenario setting*

Similarly to the modifications made to the constraints certain cost functions also need to be adapted for the scenario-based setting. In particular in (27) we need to account for $\mathcal{A}(S,F)\cup\mathcal{Q}(S,F)\cup\mathcal{Q}'(S,F)$ instead of $\mathcal{A}(S,F)\cup\mathcal{Q}(S,F)$, thus resulting in (27)′. Coherently, the cost will be denoted as $C'_v(t)$. Furthermore the cost (30) is replaced with

$$C'_{\rm asm}(t) = c^+_{\rm asm}(t)\frac{\tau}{K}\sum_{k=1}^{K} P^+_{{\rm asm},k}(t) - c^-_{\rm asm}(t)\frac{\tau}{K}\sum_{k=1}^{K} P^-_{{\rm asm},k}(t). \quad (47)$$

Finally the cost function (31) is modified to incorporate terms related to the scenario-based setting. The updated cost function, denoted as C'_y, is expressed as

$$C'_y = \sum_{i\in\mathcal{Q}(S,F)} w^+_{y,i} y^+_{\delta,i} + \sum_{i\in\mathcal{Q}(S,F)} w^-_{y,i} y^-_{\delta,i} + Q^\delta. \quad (48)$$

This adjustment accounts for additional stochastic considerations in the allocation process.

7.3. *Optimal problem for the scenario setting*

When expanding the optimization framework to include stochastic requests $\mathcal{Q}'(S,F)$ alongside with the already allocated vehicles $\mathcal{A}(S,F)$ and the deterministic requests $\mathcal{Q}(S,F)$, the former overall cost J_c in (32) is substituted with

$$J'_c = \iota_C \left[\sum_{t=S}^{F}\left[C'_v(t) + C_{\rm idm}(t) + C_{\rm dam}(t) + C'_{\rm asm}(t)\right] + C'_y\right] + \iota_Q \left[Q^a + Q^{\mathcal{D}} + Q^e + Q^\alpha + Q^P\right]. \quad (49)$$

The above cost takes into account both the modified costs due to the scenario setting and the costs related to the constraints violations. The scalars $\iota_C, \iota_Q \ge 0$ provide adjustable weighting factors, allowing the optimization to balance cost minimization and constraint adherence. The final optimization problem is formulated as follows

$$\begin{aligned}
&\min\ J'_c\\
&\text{s.t.}\\
&\text{Power variables, power rate, one station assignment } (1)' - (9)',\\
&\text{Arrival and departure time for deterministic vehicles only } (10),\\
&\text{Arrival and departure time for stochastic vehicles only } (37),\\
&\text{Unavailability for deterministic vehicles only } (11),\\
&\text{Unavailability for stochastic vehicles only } (38),\\
&\text{Already assigned vehicles } (8),\\
&\text{SOC for deterministic vehicles only } (12) - (14),\\
&\text{SOC for stochastic vehicles only } (39) - (41),\\
&\text{chargingstation constraints } (15)' - (17)',\\
&\text{Infra-day and day-ahead market variables } (18),\\
&\text{Ancillary service market variables } (19) - (21), (23),\\
&\text{Ancillary service market power } (44),\\
&\text{Power balance } (45).
\end{aligned} \quad (50)$$

8. Offline and runtime management algorithm for the scenario approach

Problem (50) can be conveniently exploited to compute the $\bar{P}^+_{\rm idm}(t)$, $\bar{P}^-_{\rm idm}(t)$, $\bar{s}^-(t)$, $\bar{s}^+(t)$ and $\bar{m}(t)$ to be used in the constraints (25) for running Algorithm 1.

These values are computed offline, the day before real-time scheduling begins, using a historical-data-driven approach. Since no booking requests have been collected at that point, both $\mathcal{Q}(S,F)$ (incoming deterministic requests) and $\mathcal{A}(S,F)$ (already allocated vehicles) are empty, while all expected future EV arrivals are treated stochastically and included in $\mathcal{Q}'(S,F)$.

Such so defined offline algorithm runs (50) and not only provides $\bar{P}^+_{\rm idm}(t)$, $\bar{P}^-_{\rm idm}(t)$, $\bar{s}^-(t)$, $\bar{s}^+(t)$ and $\bar{m}(t)$, but it is also useful in deriving, given the $a_{ij}(t)$, the trajectory $w_{ij}(t)$ to be used in problem (34) of Algorithm 1. It is important to highlight that problem (34) serves as an approximation to account for scenario realizations when handling real-time EV allocations. A more precise, but computationally more expensive, alternative would be to replace Algorithm 1 with Algorithm 2, which integrates both: the deterministic contributions from incoming vehicles in $Q(S, F)$, the already allocated vehicles in $\mathcal{A}(S, F)$, and the stochastic impact of future incoming requests in $Q'(S, F)$. Similar to Algorithm 1, Algorithm 2 is executed whenever the queue C is not empty, meaning at least one new allocation request is pending.

Algorithm 2: Process of the parking manager

Input: C, $S \in \mathbb{R}$, $F \in \mathbb{R}$
Output: $\mathcal{A}(S, F)$, $Q(S, F)$, $\bar{\alpha}_i$, $\bar{\delta}_i$
$Q(S, F) \leftarrow \emptyset$;
for $i \in C$ **do**
  $Q(S, F) \leftarrow Q(S, F) \cup \{i\}$;
  $C \leftarrow C \setminus \{i\}$;
  `//solve the optimization problem where` $\bar{P}^{+}_{\text{idm}}(t), \bar{P}^{-}_{\text{idm}}(t), \bar{s}^{-}(t), \bar{s}^{+}(t)$ `and` $\bar{m}(t)$ `are provided by the offline algorithm;`
  Solve (50) with the additional constraint (25);
  if (50) *with* (25) *is feasible* **then**
    the optimization problem provides $[\bar{\alpha}_i, \bar{\delta}_i]$;
    if EV *i accepts* $[\bar{\alpha}_i, \bar{\delta}_i]$ **then**
      $\mathcal{A}(S, F) \leftarrow \mathcal{A}(S, F) \cup \{i\}$;
    end
  else
    Notify to the EV i the allocation fail;
  end
end

9. Numerical examples

This section presents the numerical evaluation of the proposed algorithms using three test cases. Specifically, in Section 9.1 a small-scale parking case is considered, so as to grasp the proposed strategy on a reduced model. Instead, a larger parking is considered in Section 9.2. Finally, Section 9.3 provides a brief illustration of the scenario-based for generating an offline day-ahead solution accounting for future incoming EVs.

The cost profiles on the three energy markets are derived from Italian market of a random day in 2024 and provided by the Italian government GME (Gestore dei Mercati Energetici) responsible for managing and organizing the electricity and natural gas markets. For all test cases, the characteristics of the EVs are randomly generated, regardless of the parking lot size. Specifically, for each i-th EV we have:

1. the maximum incoming $z^{+,\max}_{i,\bullet}$ and outgoing $z^{-,\max}_{i,\bullet}$ power are uniformly randomly chosen in the intervals $[20, 22]$kW and $[10, 20]$kW, respectively;
2. the maximum ramp rates for the incoming and outgoing power, respectively G^{+}_{i} and G^{-}_{i}, are uniformly randomly chosen in the interval $[0.5, 1.5]$kW/min;
3. arrival and departure time α_i and δ_i are randomly selected according to a normal distribution centred, respectively at 9.00 and 17.00, both with a standard deviation of 1.5 hours;
4. the probability of an unavailability interval (we consider only at most one unavailability interval per vehicle) is chosen to be 0.2. If present, such interval has a length chosen randomly with a maximum length of the 15% of the working day associated at each EV's owner. The beginning of the unavailability interval is randomly chosen within the working day of each EV's owner;
5. charging and discharging efficiency $\eta^{+}_{i}, \eta^{-}_{i}$ are uniformly randomly chosen in the interval $[0.94, 0.99]$;
6. maximum and minimum energy that can be stored, namely $e^{\max}_{i}$ and $e^{\min}_{i}$ are uniformly randomly chosen in the intervals $[30, 60]$kW h and $[5, 10]$ kW h, respectively;
7. finally, the initial charge e_{i,α_i} is uniformly randomly chosen in the interval $[10, 50]$ % of the $e^{\max}_{i}$, while the final charge required by each EV, that is the interval $[e^{-}_{i,\delta_i}, e^{+}_{i,\delta_i}]$, is randomly selected within the interval $[10, 50]$ % of $e^{\max}_{i}$.

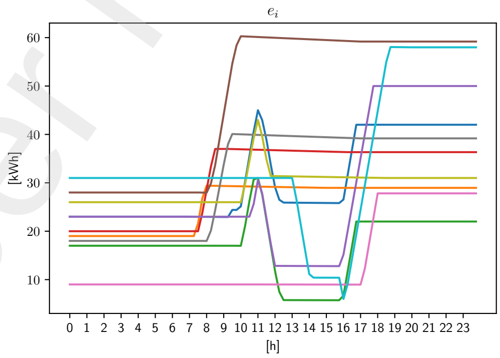


Figure 1: SOC evolution of the 10 EVs.

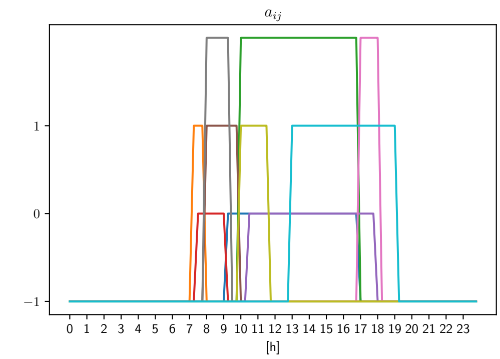


Figure 2: Allocations of the 10 EVs to the charging stations (−1 denotes no allocation).

All the simulations are conducted on a 24 hours day with sampling time of 15 minutes, then the time instant $t \in \{0, 1, \dots, 95\}$. For simplicity, the results are reported in terms of hours, but teh simulations have been performed over 96 samples.

The algorithms developed in this paper have been solved with Gurobi [31] and modeled with the Pyomo [32] a Python based open source optimization modeling language.

9.1. *Online parking management on 10 vehicles case*

To illustrate the key features of the proposed online management algorithm, here we test it on a small-scale parking problem with 10 vehicles. This setup provides a clear understanding of the algorithm's operation on a reduced model. Consistent with the proposed framework, we assume that a day-ahead solution has already been computed. The task of the online management algorithm is to optimally handle deviations from this precomputed plan as they occur in real time. As detailed in Section 5, the algorithm aims to minimize deviations from the day-ahead vehicle allocations while dynamically rescheduling power profiles as needed.

To simulate the testing scenario, we consider 3 charging stations, each with distinct characteristics generated randomly. Specifically, for station indexed with 0, $N_0 = 2$, that is two EVs can be served simultaneously, while for the remaining stations 1 and 2 we have $N_1 = 1$ and $N_2 = 1$. The maximum power $z^{+,\max}_{\bullet,j}$, with $j \in \{0, 1, 2\}$, each station can deliver is uniformly randomly chosen in the interval $[45, 75]$kW, while the maximum power $z^{-,\max}_{\bullet,j}$ each station can absorb is uniformly randomly chosen in the interval $[30, 60]$kW.

Once the 10 EVs are randomly generated and their day-ahead allocation and power profiles are computed, they are fed into the online management Algorithm 1.

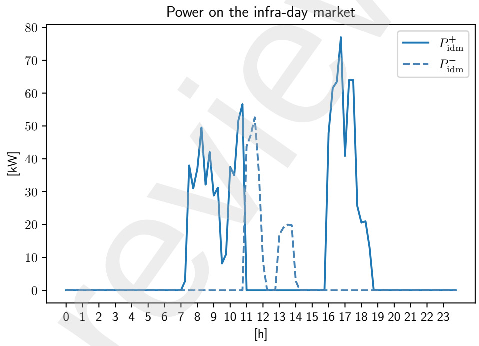
To simulate real-world uncertainties, we introduce Gaussian noise to emulate deviations between planned and real-time data. Specifically, initial and desired SOC values are modified using a Gaussian distribution with zero mean and 10% standard deviation; arrival and departure times are adjusted using a Gaussian distribution with zero mean and a standard deviation of 45 minutes; unavailability intervals are also shifted using a Gaussian distribution. With these choices, each EV can exhibit substantial deviations from its day-ahead plan, reflecting realistic operational challenges. Despite these variations, the proposed algorithm dynamically adjusts and successfully accommodates these changes.

Without loss of generality and to ease the exposition, we suppose that the progressive index of each EV is assigned according to the order it requests for allocation. That is EV 0 is the first one asking for allocation in real-time, EV 1 is the second requesting real-time allocation and so on until EV 9, which is the last one requesting allocation.

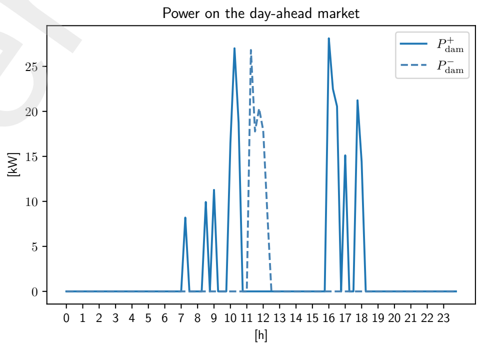
Fig. 1 and Fig. 2 depict the SOC and the allocations (the value -1 is used to denote no allocation, while values $0, 1, 2$ are referred to the indexes of the three charging stations) of all the EVs for the online case, respectively. As it is possible to see, the proposed management algorithm is able to allocate all the EVs. The power bought and sold on the three markets are reported in Fig. 3.

Further, the comparison with the day-ahead solution is here reported through a series of selected pictures.

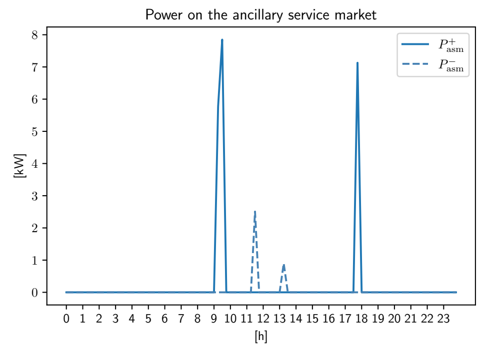
Specifically, Fig. 4 and Fig. 5 report the comparison of the total allocation (that is, how many vehicles are allocated regardless of which charging station) and the total incoming and outgoing power. As it is possible to see, despite the fact that the online case is generated with significant perturbations of



(a)



(b)



(c)

Figure 3: Bought ($^+$) and sold ($^-$) powers on the three markets: (a) infra-day; (b) day-ahead; (c) ancillary service for the 10 EVs case.

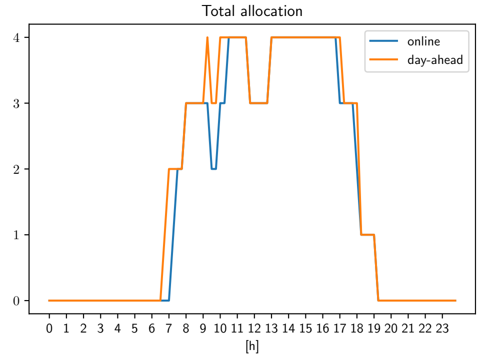
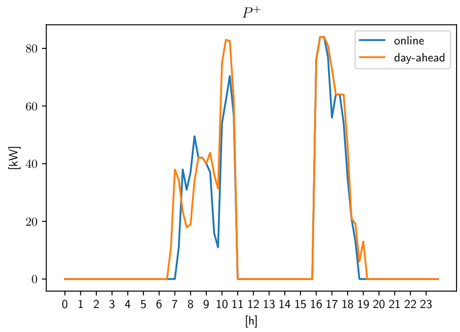
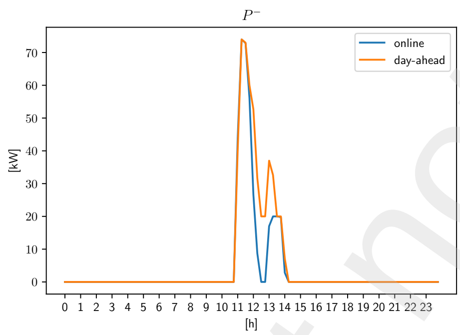


Figure 4: Comparison between online (blue) and day-ahead computed (orange) total parking allocation for the 10 EVs case.



(a)



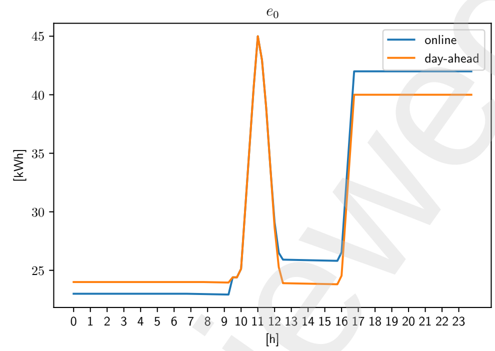
(b)

Figure 5: Bought (a) and sold (b) total power comparison between online (blue) and day-ahead computed (orange) solutions for the 10 EVs case.

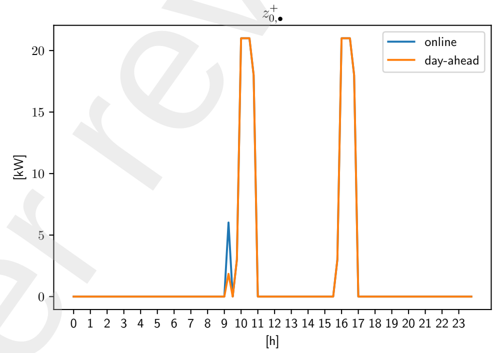
the data used to compute a day-ahead solution, the management algorithm try to adhere as much as possible to the planned vehicles allocated time slots, while modifying (if needed) the power profiles.

To further highlight the ability of the proposed management algorithm for finding online solutions, we decided to show the comparison between online and offline cases of two representative EVs, namely EV 0 and EV 4 (the first and the middle one asking for allocation).

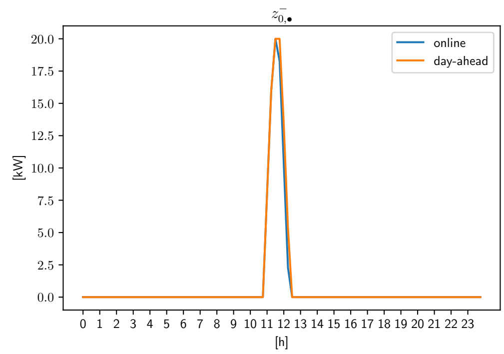
They are represented in Fig. 6 and Fig. 7, respectively where the SOC (a), incoming $z^+_{i,\bullet}$ (b) and outgoing $z^-_{i,\bullet}$ (c) power profiles are shown.



(a)



(b)

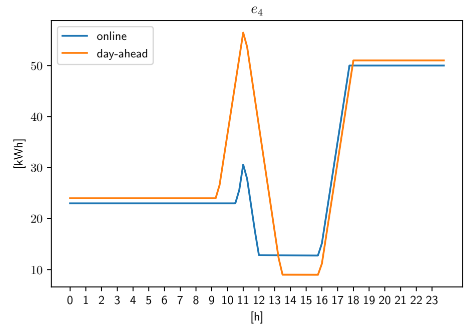


(c)

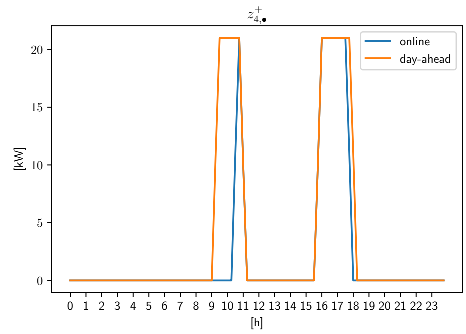
Figure 6: Comparison between the online (blue) and day-ahead (orange) solutions for EV 0 for the 10 EVs case: (a) SOC; (b) incoming power $z^+_{0,\bullet}$; outgoing power $z^-_{0,\bullet}$.

Further, as showed in the paper, our algorithm recomputes at any new allocation request issued by later EVs the power profiles for the EVs already allocated (the scheduled allocation does not change).

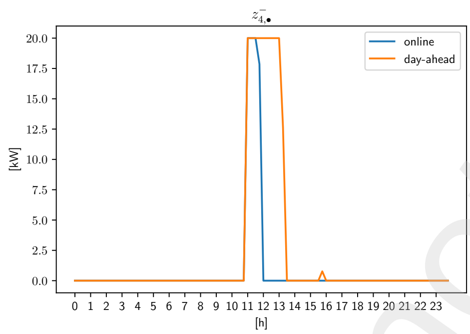
For the representative EV 0 and EV 4 this is shown in Fig. 8 and Fig. 9, respectively, where the subsequent SOC is shown. In particular, since EV 0 is the first one asking for allocation, it is subjected to other nine reschedules (each one triggered by a new incoming EV). The last reschedule (shot 9) is the one corresponding to the actual SOC profile (also reported in Fig. 6(a) in blue) since no more re-computations will be performed.



(a)



(b)



(c)

Figure 7: Comparison between the online (blue) and day-ahead (orange) solutions for EV 4 for the 10 EVs case: (a) SOC; (b) incoming power $z^+_{4,\bullet}$; outgoing power $z^-_{4,\bullet}$.

With the same argument, being EV 4 the fifth vehicle asking for an allocation, it will be subjected to only five reschedules, as shown in Fig. 9. Also in this case, the last reschedule (shot 9) corresponds to the actual SOC trajectory.

9.2. *Online parking management on 90 vehicles case*

We show the effectiveness of the proposed strategy on a bigger and more realistic case where we consider 90 EVs asking for allocation and a parking corporate with 60 recharge stations. All the stations are equal and can serve only one EV at the time. This is equivalent to consider, from a modeling point of view, a unique station with up to 60 contemporary recharges. Data are randomly generated as described in the introduction to this section.

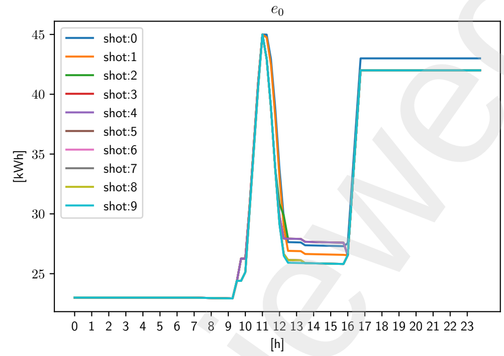


Figure 8: Online computations of the SOC solutions for EV 0 at any new allocation request for the 10 EVs case.

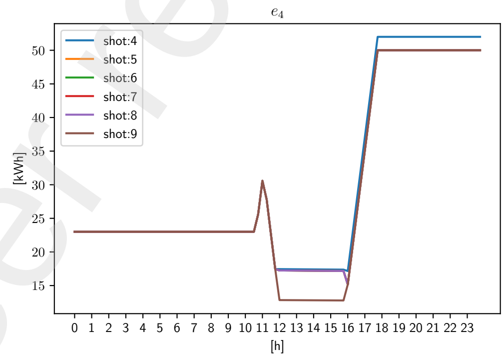


Figure 9: Online computations of the SOC solutions for EV 4 at any new allocation request for the 10 EVs case.

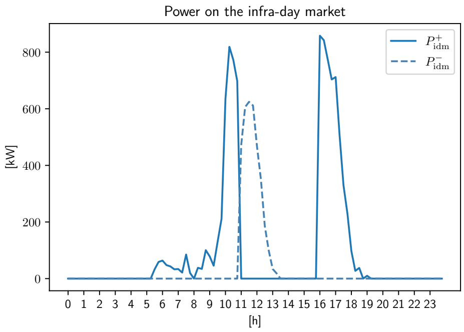
We report the solution for input and output power on the three markets in Fig. 10. As it is possible to see, the management parking is able to satisfy the online requests of all the EVs. The time to compute a solution for each vehicle request takes around 6s on an Windows Intel i7 equipped machine, compatible with an online service request. Notice that, since the boolean variables need to be decided only for the vehicle asking for allocation at a given time, their number is equal to 9 (the time horizon) for each optimization problem instance.

Further, we compare the online solution with respect to a previously computed day-ahead solution on the nominal data. Despite the latter are subjected to significant perturbation on the online case, the management algorithm find a solution whose allocations adhere reasonably to the day-ahead case, as clearly illustrated in Fig. 11, where the percentage of the total occupancy (100% corresponds to all the 60 lots simultaneously engaged) is compared. Further, comparison of the total power is shown in Fig. 12.

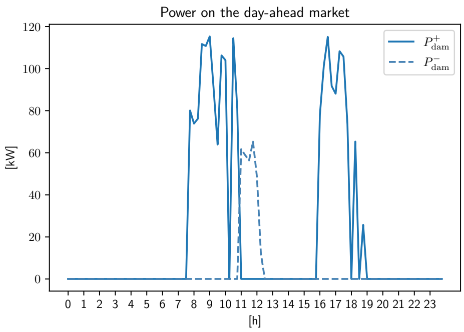
We show also the SOC comparison for EV 44 taken as a representative example (it is the one requiring allocation after half EVs). The comparison is reported in Fig. 13.

9.3. *Scenario-based offline solution generation*

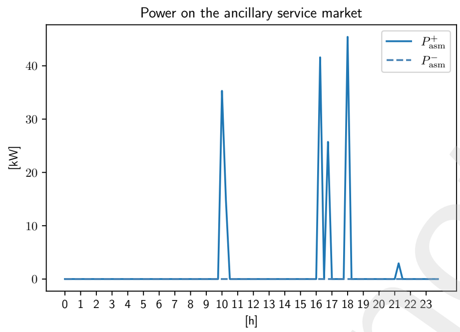
To conclude the numerical illustration, in this subsection we report a simulation for the scenario based approach developed



(a)



(b)



(c)

Figure 10: Bought ($^+$) and sold ($^-$) powers on the three markets: (a) infra-day; (b) day-ahead; (c) ancillary service for the 90 EVs case.

in Section 7. Specifically, we consider the same setting described in Section 9.2 (90 EVs and 60 equal charging stations). We consider the optimization problem (50) where all the 90 vehicles are involved, thus generating a day-ahead offline solution on prevision data. The data of the EVs are generated randomly as already described at the beginning of this section. On such data, we construct ten scenarios and optimize over all of them as we described above. More in details, for each vehicle we randomly change the arrival and departure time with a zero mean Gaussian distribution of standard deviation of 45 minutes. The same standard deviation is used to randomly shift the unavailability interval. In addition to this, for each vehicle the initial SOC and final SOC interval boundaries are modified in the scenario generation through a zero mean Gaussian distri-

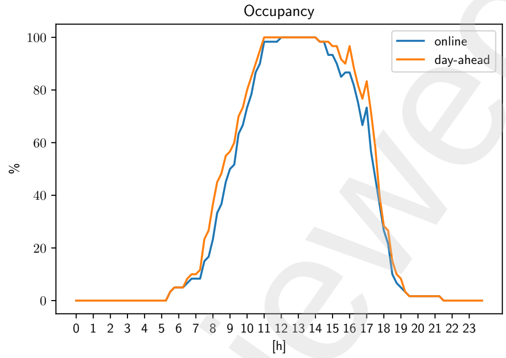
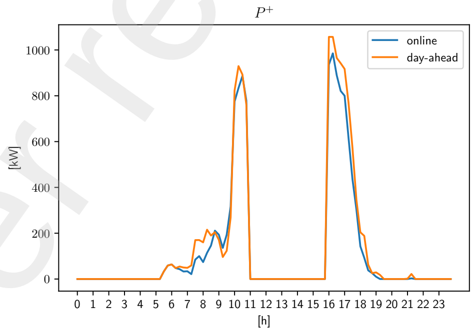
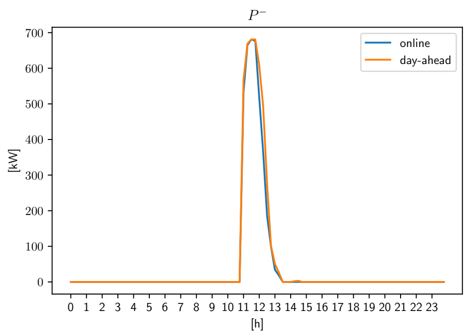


Figure 11: Comparison between online (blue) and day-ahead computed (orange) total parking percentage allocation for the 90 EVs case.



(a)



(b)

Figure 12: Bought (a) and sold (b) total power comparison between online (blue) and day-ahead computed (orange) solutions for the 90 EVs case.

bution with a 10% standard deviation. Finally, the downward and upward signals $\omega^-(t)$, $\omega^+(t)$ are generated randomly for the ten scenarios via modifying a given signal (we consider a historical trajectory as basis) with a zero mean and 10% standard deviation Gaussian distribution.

We deemed the ten scenarios generated in this way sufficiently rich and diverse to be adopted for optimization (50). The solution of such problem provides an allocation of all the vehicles across all the scenarios.

In Fig. 14 we report the total allocation (in percentage) of the whole parking, while the incoming and outgoing power over the day ahead market is reported in Fig. 15(a). Finally, Fig. 15(b)

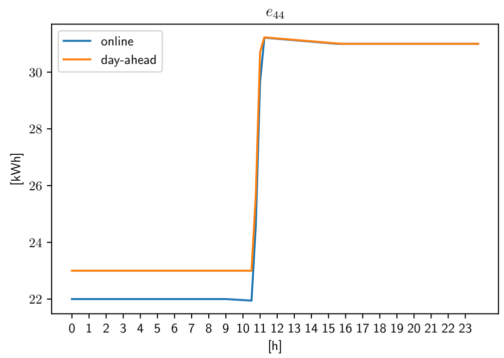


Figure 13: Comparison between the online (blue) and day-ahead computed (orange) SOC for EV 44 for the 90 EVs case.

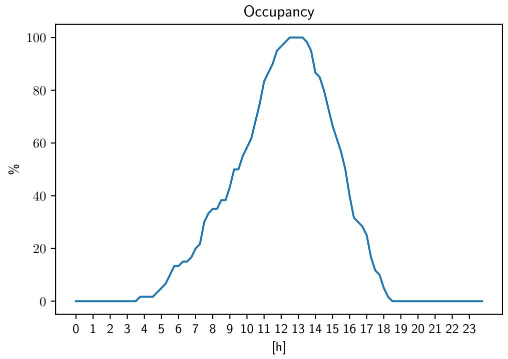


Figure 14: Total parking percentage allocation for the 90 EVs scenario based case.

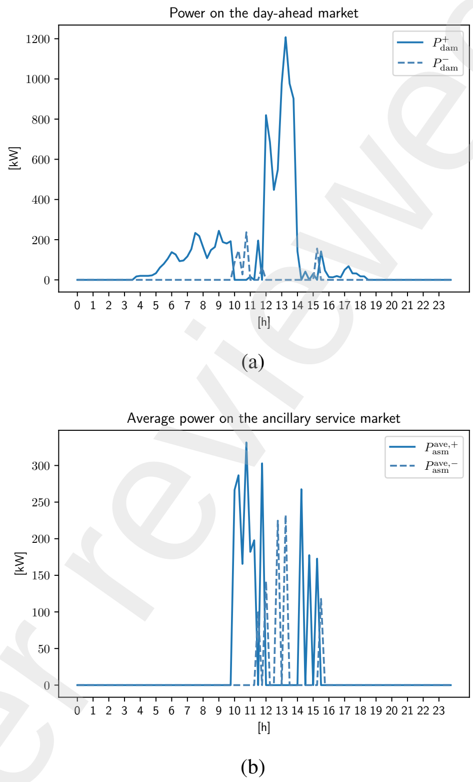


Figure 15: Bought ($^+$) and sold ($^-$) powers for the 90 EVs scenario based case: (a) on the day-ahead market; (b) average power on the ancillary service market.

reports the average power trajectory on the ancillary service market for the scenario considered, both incoming and outgoing.

To reduce the number of figures, we do not report the incoming and outgoing power profiles for the single vehicles. Nevertheless, we outline that, as explained in Section 7, the scenario solution can play the role of the day ahead solution that the management algorithm tries to adhere in real-time (that is a data for the online problem), as illustrated in Section 9.1 and Section 9.2.

10. Conclusions

This paper proposes a novel parking management system for a company which offers to the employees the possibility of recharge their EVs. The designed management algorithm is specifically suited for runtime settings where charging occurs during work hours. Employees can book charging slots, specifying arrival and departure time, desired final charge level, and unavailable time slots.

The core of the system is a mixed integer linear program approach that optimizes charging schedules while accounting for various uncertainties, including arrival and departure times and initial SOC. The system supports the participation in energy markets and accommodates also a reduced QoS proposals when necessary. Furthermore it addresses real-word challenges, such as asynchronous booking requests and stochastic in EV behavior. By optimizing resource allocation and energy management, the system ensures that each EV reaches its desired SOC by the end of the parking session while maximizing an objective function related to parking operations and energy market participation.

The solution proposed in the paper offers a comprehensive, adaptable, and practical framework for online EVs charging scheduling in company parking lots, effectively overcoming the limitations of previous works and catering to the real-world complexities of EVs charging management. This system is designed for direct deployment in company parking lots and can be implemented as a service. It manages both booking requests and forecasted data as well as realistic constraints of human customers, such as desired SOC, time availability and the fact that, once fixed, an allocation cannot be changed (however power profiles can be rescheduled),while considering human-related constraints such as desired SOC, time availability, and the inability to modify allocations once assigned (though power profiles can still be rescheduled). It also accounts for infrastructure constraints, such as heterogeneous charging station capabilities and a limited number of charging stations compared to the number of EVs requiring charging. Moreover, the proposed algorithm simultaneously solves a scheduling and power management problem, operating in a dynamic setting that allows it to respond to new requests in real time and re plan accordingly.

In this appendix we show that the optimization problem (35) naturally excludes the possibility of having at the same time in and out power flows, both from/to the grid and the vehicles. We will set the discussion without being to formal but informative enough for the reader so as to grasp the reasons behind this fact.

Let us suppose by contradiction an optimal solution such that $P^{+,*}_{\rm idm}(t) > 0$ and $P^{-,*}_{\rm idm}(t) > 0$ at a certain time t.

Problem (35) can be modified in an equivalent way introducing the decision variable $P^{-+}_{\rm idm}(t)$, the constraint

$$P^{-+}_{\rm idm}(t) = P^{-}_{\rm idm} - P^{+}_{\rm idm}, \quad (.1)$$

and modifying (24a) as

$$\sum_{i\in\mathcal{A}(S,F)\cup\mathcal{Q}(S,F)} \left[z^{+}_{i,\bullet}(t) - z^{-}_{i,\bullet}(t)\right] + P^{-+}_{\rm idm}(t) - P^{+}_{\rm dam}(t) + P^{-}_{\rm dam}(t) - P^{+}_{\rm asm}(t) + P^{-}_{\rm asm}(t) = 0. \quad (.2)$$

$\forall t \in [S, F]$. We can easily show that there exists a feasible solution that keeps null one variable among $P^{+}_{\rm idm}(t)$ and $P^{-}_{\rm idm}(t)$ yielding a cost value lower than the one provided by that with variables $P^{+,*}_{\rm idm}(t), P^{-,*}_{\rm idm}(t)$ both strictly positive. Indeed, if $P^{-+,*}_{\rm idm}(t) \leq 0$, the solution with $\hat{P}^{+}_{\rm idm}(t) = -P^{-+,*}_{\rm idm}(t)$ and $\hat{P}^{-}_{\rm idm}(t) = 0$ is still feasible because satisfies constraints (.1) and (.2). Furthermore, the cost term $c^{+}_{\rm idm}\tau\hat{P}^{+}_{\rm idm}(t)$ provided by this solution is lower than $c^{+}_{\rm idm}\tau P^{+,*}_{\rm idm}(t) - c^{-}_{\rm idm}\tau P^{-,*}_{\rm idm}(t)$ provided by the original solution. The latter, indeed, can be rewritten as $c^{+}_{\rm idm}\tau[\hat{P}^{+}_{\rm idm}(t) + P^{-,*}_{\rm idm}(t)] - c^{-}_{\rm idm}\tau P^{-,*}_{\rm idm}(t)$ which provides the additional positive cost $(c^{+}_{\rm idm} - c^{-}_{\rm idm})\tau P^{-,*}_{\rm idm}(t)$.

The same reasoning can be done if $P^{-+,*}_{\rm idm}(t) > 0$. In this case the choice $\hat{P}^{+}_{\rm idm}(t) = 0$ and $\hat{P}^{-}_{\rm idm}(t) = P^{-+,*}_{\rm idm}(t)$ satisfies the problem constraints and yields to a cost function $-c^{-}_{\rm idm}\tau\hat{P}^{-}_{\rm idm}(t)$. The cost term provided by the original solution can be rewritten as $(c^{+}_{\rm idm} - c^{-}_{\rm idm})\tau P^{+,*}_{\rm idm}(t) - c^{-}_{\rm idm}\tau\hat{P}^{-}_{\rm idm}(t)$ again providing an additional cost term.

Being the solution that keeps at the same time one of the two powers null feasible and better than the original one, the latter is not optimal. This contradicts its optimality hypothesis.

The exact same reasoning can be applied for the powers on the day-ahead market and ancillary service market or combinations among them.

To exclude the possibility of having an optimal solution that requires chargingand discharging at the same time a vehicle i we can conduct a similar reasoning, although slightly more complex. Again, by contradiction, let us suppose there exists an optimal solution with $z^{+,*}_{i,\bullet}(t) > 0$ and $z^{-,*}_{i,\bullet}(t) > 0$ for some t. Let us also introduce the new decision variable $u_i(t)$ with the constraint

$$u_i(t) = \eta^{+}_{i} z^{+}_{i,\bullet}(t) - \frac{1}{\eta^{-}_{i}} z^{-}_{i,\bullet}(t). \quad (.3)$$

The optimization problem can be equivalently written considering this additional decision variable, constraint (.3) and modifying (12) as

$$e_i(t+1) = \lambda_i e_i(t) + \tau u_i(t), \quad (.4)$$

$\forall i \in \mathcal{A}(S, F) \cup \mathcal{Q}(S, F)$. If $u^{*}_{i}(t) > 0$ we can consider the candidate solution of the optimization problem that differs from the original one in having the new powers entering and leaving vehicle i respectively as $\hat{z}^{+}_{i,\bullet}(t) > 0$ and $\hat{z}^{-}_{i,\bullet}(t) = 0$ such that $u^{*}_{i}(t) = \eta^{+}_{i}(t)\hat{z}^{+}_{i,\bullet}(t)$. Let us call $\tilde{z}^{+}_{i,\bullet}(t) = z^{+}_{i,\bullet}(t) - \hat{z}^{+}_{i,\bullet}(t)$. We obviously have that $\eta^{+}_{i}\tilde{z}^{+}_{i,\bullet}(t) = \frac{1}{\eta^{-}_{i}} z^{-,*}_{i,\bullet}(t)$.

From this, it is possible to rewrite the cost contribution of the original solution $c^{-}_{v}\tau z^{-,*}_{i,\bullet} - c^{+}_{v}\tau z^{+,*}_{i,\bullet}$ as $c^{-}_{v}\tau z^{-,*}_{i,\bullet} - c^{+}_{v}\tau[\hat{z}^{+}_{i,\bullet}(t) + \tilde{z}^{+}_{i,\bullet}(t)]$ which in turns is equal to $(c^{-}_{v}\eta^{-}_{i}\eta^{+}_{i} - c^{+}_{v})\tau\tilde{z}^{+}_{i,\bullet}(t) - c^{+}_{v}\hat{z}^{+}_{i,\bullet}(t)$. By remembering (26), the term $c^{-}_{v}\eta^{-}_{i}\eta^{+}_{i} - c^{+}_{v}$ is strictly positive. Therefore, the cost associated to the original solution is higher than the one ($-c^{+}_{v}\hat{z}^{+}_{i,\bullet}(t)$ only) of the proposed candidate solution.

However, it is worth to highlight that the candidate solution does not respect constraint (24a). Indeed, the term $z^{+,*}_{i,\bullet} - z^{-,*}_{i,\bullet} = \hat{z}^{+}_{i,\bullet} + \tilde{z}^{+}_{i,\bullet} - z^{-,*}_{i,\bullet}$ is replace by the solely term $\hat{z}^{+}_{i,\bullet}$. The missing term $\tilde{z}^{+}_{i,\bullet} - z^{-,*}_{i,\bullet}$ is a positive power entering into the vehicle that need to be balanced for the new candidate solution via reducing the imported power from the three markets. This further translates in an additional reduction of the overall cost.

A similar reasoning can be done in case $u^{*}_{i}(t) \leq 0$. Indeed, the candidate solution $\hat{z}^{+}_{i,\bullet}(t) = 0$ and $\hat{z}^{-}_{i,\bullet}(t) \geq 0$ such that $u^{*}_{i}(t) = -\eta^{-}_{i}(t)\hat{z}^{-}_{i,\bullet}(t)$ yields to a cost $c^{-}_{v}\tau\hat{z}^{-}_{i,\bullet}(t)$ which is lower than $c^{-}_{v}\tau\hat{z}^{-}_{i,\bullet}(t) + (c^{-}_{v}\eta^{+}_{i}\eta^{-}_{i} - c^{+}_{v})\tau z^{+,*}_{i,\bullet}(t)$ provided by the original solution. Also, to satisfy constraint (24a) the additional incoming power term $z^{+,*}_{i,\bullet}(t) - \tilde{z}^{-}_{i,\bullet}(t)$ provided by the original solution must be compensated by a reduction in imported power, further lowering the overall cost.

Therefore, irrespective of the sign of $u^{*}_{i}(t)$, there always exists a solution with one of the two terms among $z^{+}_{i,\bullet}(t), z^{-}_{i,\bullet}(t)$ null and a reduced imported power which is feasible and provides a lower cost with respect to a solution that keeps both the terms $z^{+}_{i,\bullet}(t), z^{-}_{i,\bullet}(t)$ strictly positive. In view of this, the hypothesis of being the latter an optimal solution is contradicted.

References

[1] R. R. Kumar, K. Alok, Adoption of electric vehicle: A literature review and prospects for sustainability, Journal of Cleaner Production 253 (2020) 119911.

[2] T. Lin, H. Rivano, F. Le Mouël, A survey of smart parking solutions, IEEE Transactions on Intelligent Transportation Systems 18 (12) (2017) 3229–3253.

[3] V. C. Onishi, C. H. Antunes, J. P. F. Trovão, Optimal energy and reserve market management in renewable microgrid-PEVs parking lot systems: V2G, demand response and sustainability costs, Energies 13 (8) (2020) 1884.

[4] P. V. Dahiwale, Z. H. Rather, I. Mitra, A comprehensive review of smart charging strategies for electric vehicles and way forward, IEEE Transactions on Intelligent Transportation Systems 25 (9) (2024) 10462–10482.

[5] M. Ebrahimi, M. Shafie-Khah, H. Laaksonen, Uncertainty-observed virtual battery model for an electric vehicle parking lot enabling charger-sharing modelling, Journal of Energy Storage 89 (2024) 111578.

[6] Q. Wang, C. Huang, C. Wang, K. Li, N. Xie, Joint optimization of bidding and pricing strategy for electric vehicle aggregator considering multi-agent interactions, Applied Energy 360 (2024) 122810.

[7] I. A. Zenhom, M. F. Shaaban, W. A. Omran, Grid interactive charging of EVs in PV-powered parking lots considering uncertainties, IEEE Access 11 (2023) 111292–111301.

[8] W. Su, M.-Y. Chow, Performance evaluation of an EDA-based large-scale plug-in hybrid electric vehicle charging algorithm, IEEE Transactions on Smart Grid 3 (1) (2011) 308–315.

[9] H. Farzin, M. Fotuhi-Firuzabad, M. Moeini-Aghtaie, Reliability studies of modern distribution systems integrated with renewable generation and parking lots, IEEE Transactions on Sustainable Energy 8 (1) (2016) 431–440.

[10] M. Shariatzadeh, C. Henggeler Antunes, M. A. Lopes, Charging scheduling in a workplace parking lot: Bi-objective optimization approaches through predictive analytics of electric vehicle users' charging behavior, Sustainable Energy, Grids and Networks 39 (2024) 101463.

[11] Z. Liu, P. Li, J. Liu, X. He, J. Han, C. Zi, N. Liang, A day-ahead energy management and reserve declaration strategy for parking lot based on electric vehicle accessing guidance, International Journal of Electrical Power & Energy Systems 136 (2022) 107704.

[12] R. Bernal, D. Olivares, M. Negrete-Pincetic, Á. Lorca, Management of EV charging stations under advance reservations schemes in electricity markets, Sustainable Energy, Grids and Networks 24 (2020) 100388.

[13] G. S. Fernandez, V. Krishnasamy, S. Kuppusamy, J. S. Ali, Z. M. Ali, A. El-Shahat, S. H. Abdel Aleem, Optimal dynamic scheduling of electric vehicles in a parking lot using particle swarm optimization and shuffled frog leaping algorithm, Energies 13 (23) (2020) 6384.

[14] F. Giordano, C. Diaz-Londono, G. Gruosso, Comprehensive Aggregator Methodology for EVs in V2G Operations and Electricity Markets, IEEE Open Journal of Vehicular Technology 4 (2023) 809–819. doi:10.1109/OJVT.2023.3323087.

[15] D. Hao, X. Ren, A. S. Mohammed, Optimal design of electric vehicle parking lot based on energy management considering hydrogen storage system and demand side management, Journal of Energy Storage 42 (2021) 103045.

[16] M. A. Gharibi, H. Nafisi, H. Askarian-abyaneh, A. Hajizadeh, Deep learning framework for day-ahead optimal charging scheduling of electric vehicles in parking lot, Applied Energy 349 (2023) 121614.

[17] S. S. Karimi Madahi, H. Nafisi, H. Askarian Abyaneh, M. Marzband, Co-optimization of energy losses and transformer operating costs based on smart charging algorithm for plug-in electric vehicle parking lots, IEEE Trans. Transportation Electrification 7 (2) (2021) 527–541.

[18] L. Pilotti, L. Moretti, E. Martelli, G. Manzolini, Optimal E-fleet charging station design with V2G capability, Sustainable Energy, Grids and Networks 36 (2023) 101220.

[19] D. Jakus, J. Vasilj, D. Jolevski, Optimal electric vehicle parking lot energy supply based on mixed-integer linear programming, Energies 16 (23) (2023) 7793.

[20] M. Aicardi, V. Casella, G. Ferro, R. Minciardi, L. Parodi, M. Robba, Optimal control of electric vehicles charging in a smart parking, IFAC-PapersOnLine 55 (5) (2022) 66–71.

[21] Á. Porras, R. Fernández-Blanco, J. M. Morales, S. Pineda, An efficient robust approach to the day-ahead operation of an aggregator of electric vehicles, IEEE Transactions on Smart Grid 11 (6) (2020) 4960–4970.

[22] G. R. C. Mouli, M. Kefayati, R. Baldick, P. Bauer, Integrated PV charging of EV fleet based on energy prices, V2G, and offer of reserves, IEEE Transactions on Smart Grid 10 (2) (2017) 1313–1325.

[23] M. Honarmand, A. Zakariazadeh, S. Jadid, Integrated scheduling of renewable generation and electric vehicles parking lot in a smart microgrid, Energy Conversion and Management 86 (2014) 745–755.

[24] G. J. Osório, M. Lotfi, M. Gough, M. Javadi, H. M. Espassandim, M. Shafie-khah, J. P. Catalao, Modeling an electric vehicle parking lot with solar rooftop participating in the reserve market and in ancillary services provision, Journal of Cleaner Production 318 (2021) 128503.

[25] U. B. Irshad, M. S. H. Nizami, S. Rafique, M. J. Hossain, S. C. Mukhopadhyay, A battery energy storage sizing method for parking lot equipped with EV chargers, IEEE Systems Journal 15 (3) (2020) 4459–4469.

[26] R. Vignali, L. Pellegrino, A. Gatti, S. Canevese, An investigation of V2G profitability with robust optimization and bidding price heuristics, in: 2020 AEIT International Annual Conference (AEIT), 2020, pp. 1–6.

[27] R. Vignali, A. Falsone, F. Ruiz, G. Gruosso, Towards a comprehensive framework for V2G optimal operation in presence of uncertainty, Sustainable Energy, Grids and Networks 31 (2022) 100740.

[28] G. C. Calafiore, M. C. Campi, The scenario approach to robust control design, IEEE Transactions on Automatic Control 51 (5) (2006) 742–753.

[29] S. Kim, R. Pasupathy, S. G. Henderson, A guide to sample average approximation, Handbook of Simulation Optimization (2015) 207–243.

[30] F. Verrilli, A. Parisio, L. Glielmo, Stochastic model predictive control for optimal energy management of district heating power plants, in: 2016 IEEE 55th Conference on Decision and Control (CDC), IEEE, 2016, pp. 807–812.

[31] Gurobi Optimization, LLC, Gurobi Optimizer Reference Manual (2024).
URL `https://www.gurobi.com`

[32] M. L. Bynum, G. A. Hackebeil, W. E. Hart, C. D. Laird, B. L. Nicholson, J. D. Siirola, J.-P. Watson, D. L. Woodruff, Pyomo–optimization modeling in python, 3rd Edition, Vol. 67, Springer Science & Business Media, 2021.