Modeling and control of fully-actuated MRAVs

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Half-day Tutorial Session at ICUAS 2026 (09:00 – 13:00), 15th June 2026
Room Calypso A – Divani Corfu Palace

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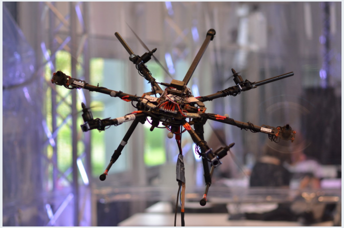
Multi-Rotor Aerial Vehicles (MRAVs) – Recall

Under-actuated MRAVs (u-MRAVs)

- Controllable DoFs: 4 (3 position, 1 rotation),
- Wide spread; commercially viable since 2010,
- Balance between capability and simplicity; differentially flat system.

Fully-actuated MRAVs (f-MRAVs)



- Controllable DoFs: 6 (3 position, 3 rotation),
- Mainly subject of research: impedance control, force application,
- Wide range of designs with both fixed and tiltable propellers.

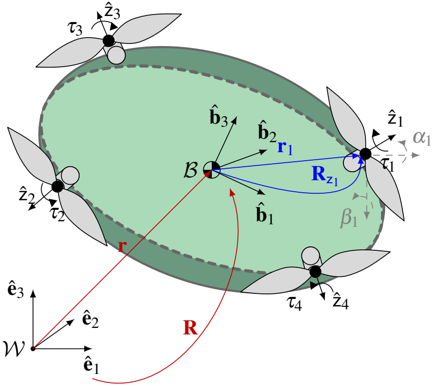
f-MRAVs

Key characteristics

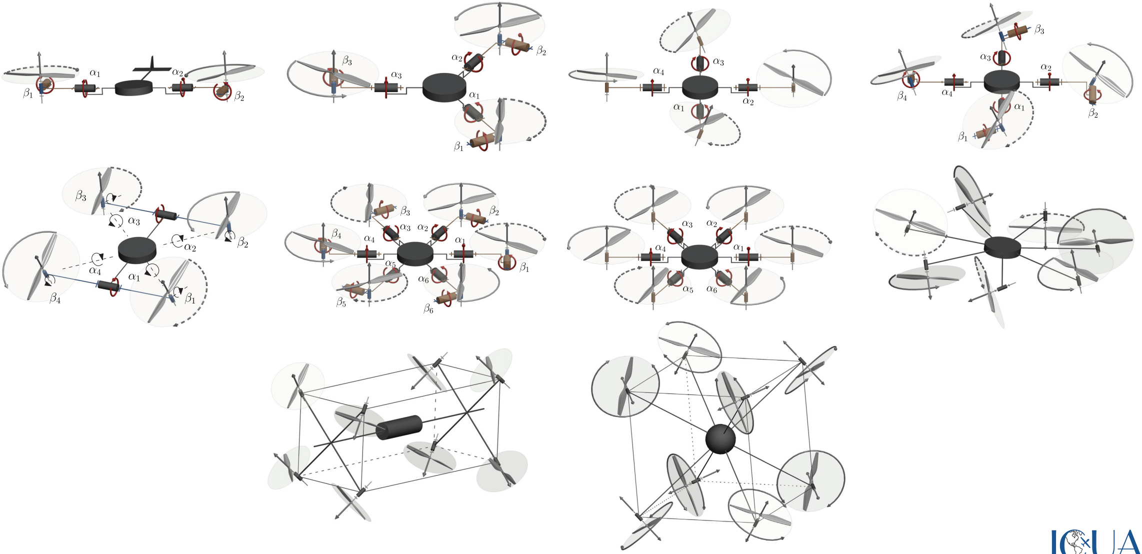
- ≥ 6 rotors with non-parallel thrust directions (full 6-DoFs actuation),
- Decoupled position and orientation control,
- Enables hovering with arbitrary attitude.

Representative configurations

- Tilted-rotor hexarotors and octorotors,
- Omnidirectional platforms (e.g., Voliro, OMR),
- Mechanically simple or overactuated designs,
- Suitable for aerial manipulation and inspection.



Popular f-MRAVs frame configurations



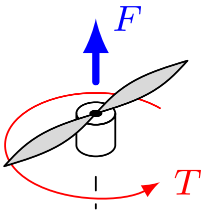
Single propeller model – Recall

Propeller thrust model (simplified)

Thrust is produced due to propellers' lift

$$F \approx k\omega^2$$

- F: thrust force [N],
- k: linear coefficient [$\mathrm{N\,s^2\,rad^{-2}}$],
- ω: propeller rate [$\mathrm{rad\,s^{-1}}$].



Motor dynamics (closed loop)

$$\dot{\omega} = -\frac{1}{\tau_m}(\omega - \omega_d)$$

- τ_m: time constant, $\approx 30\,\mathrm{ms}$,
- ω: propeller rate [$\mathrm{rad\,s^{-1}}$],
- ω_d: desired propeller rate [$\mathrm{rad\,s^{-1}}$].

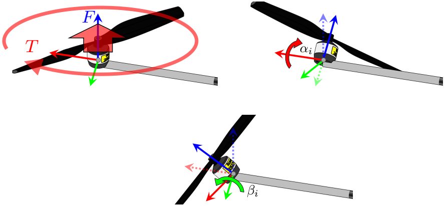
Propeller torque model (simplified)

Torque is produced due to propellers' drag

$$T \approx c_t F$$

- F: thrust force [N],
- c_t: linear torque coefficient [m].

f-MRAV dynamics model



- Each rotor generates a thrust vector $\mathbf{F}_i$ oriented along a tilted direction,
- Rotor position $\mathbf{r}_i$ is expressed in the body frame $\mathcal{B}$,
- Rotor orientation is described by vector $\hat{\mathbf{z}}_i = \{\alpha_i, \beta_i, \gamma_i\}$,
- Resulting total force $\mathbf{F}_t = \sum_i \mathbf{F}_i$,
- Resulting total torque $\boldsymbol{\tau}_t = \sum_i \mathbf{r}_i \times \mathbf{F}_i + \mathbf{T}_i$, where $\mathbf{T}_i$ denotes the aerodynamic drag torque generated by rotor i

Force-Torque allocation

$$\begin{bmatrix} \mathbf{F}_t \\ \boldsymbol{\tau}_t \end{bmatrix} = \boldsymbol{\Gamma} \begin{bmatrix} F_1 \\ \vdots \\ F_n \end{bmatrix}, \quad \text{with} \quad \boldsymbol{\Gamma} = \begin{bmatrix} \hat{\mathbf{z}}_1 & \cdots & \hat{\mathbf{z}}_n \\ \mathbf{r}_1 \times \hat{\mathbf{z}}_1 + \mathbf{T}_1 & \cdots & \mathbf{r}_n \times \hat{\mathbf{z}}_n + \mathbf{T}_n \end{bmatrix} \quad \Rightarrow \quad \boldsymbol{\Gamma} \in \mathbb{R}^{6\times n}, \ \mathrm{rank}(\boldsymbol{\Gamma}) = 6$$

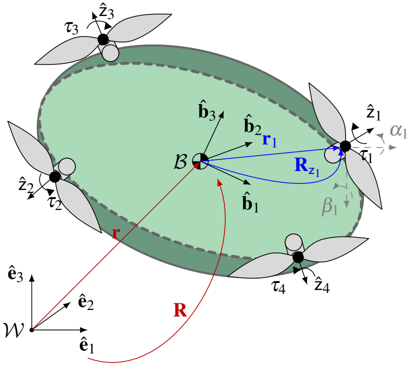
The system **can produce any desired wrench** in 6-dimensional space (3D force + 3D torque)

f-MRAV dynamics

Newton-Euler formulation

$$\begin{cases} \dot{\mathbf{r}}^{\mathcal{W}} = \mathbf{v} \\ \dot{\boldsymbol{\eta}} = \mathbf{T}(\boldsymbol{\eta})\boldsymbol{\zeta}^{\mathcal{B}} \\ m\dot{\mathbf{v}} = -m\mathbf{g}^{\mathcal{W}} + \mathbf{R}(\boldsymbol{\eta})\mathbf{F}(\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\gamma})\mathbf{u} \\ \mathbf{J}\dot{\boldsymbol{\zeta}}^{\mathcal{B}} = -\boldsymbol{\zeta}^{\mathcal{B}} \times \mathbf{J}\boldsymbol{\zeta}^{\mathcal{B}} + \mathbf{M}(\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\gamma})\mathbf{u} \end{cases}, \quad \mathbf{u} = \begin{bmatrix} \omega_1 \\ \vdots \\ \omega_n \end{bmatrix}$$

- $\mathbf{r}^{\mathcal{W}}$: position vector [m]; m: mass [kg],
- $\mathbf{g}$: gravity vector $[0, 0, -9.8]^\top$ $[\mathrm{m\,s^{-2}}]$,
- $\mathbf{J}$: moment of inertia $\in \mathbb{R}^{3\times3}$ $[\mathrm{kg\,m^2}]$,
- $\boldsymbol{\zeta}$: angular velocity; $\mathbf{T}, \mathbf{R}$: rotation matrices $\in \mathbb{R}^{3\times3}$,
- $\mathbf{M} \in \mathbb{R}^{3\times n}$: maps propeller rates to body torques,
- $\mathbf{F} \in \mathbb{R}^{3\times n}$: maps propeller rates to body forces.



Dynamic modeling of omnidirectional MRAVs

Extension of Newton-Euler equations

Rotor directions $\hat{\mathbf{z}}_i(t)$ and **positions** $\mathbf{r}_i(t)$ **vary over time** due to active tilting, leading to time-varying expressions:

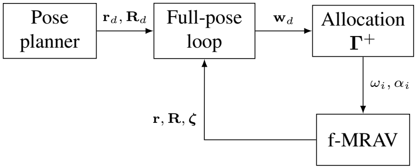
$$\mathbf{F}_t(t) = \sum_{i=1}^{n} k\omega_i^2(t)\,\hat{\mathbf{z}}_i(t),$$

$$\boldsymbol{\tau}_t(t) = \sum_{i=1}^{n} \mathbf{r}_i(t) \times \left(k\omega_i^2(t)\hat{\mathbf{z}}_i(t)\right) + c_t\omega_i^2(t)\,\hat{\mathbf{z}}_i(t).$$

Consequences

- $\boldsymbol{\Gamma}(t)$ becomes **time-varying**, **nonlinear**, and **state-dependent**,
- **Control inputs:** $\omega_i(t)$ and tilting angles $\alpha_i(t)$, $\beta_i(t)$,
- More actuators allow full-pose tracking and lateral force control without reorienting the platform.

f-MRAV control: from cascade to unified



What changes with respect to u-MRAVs?

- Translational and rotational dynamics are **decoupled** at the wrench level $\Rightarrow$ no need to tilt the body to translate,
- Position $\mathbf{r}$ and orientation $\mathbf{R}$ can be tracked **independently** on $\mathrm{SE}(3)$,
- Differential flatness is **lost in general**: the system is not flat in $(\mathbf{r}, \psi)$ – the full pose is the natural output,
- Allocation $\mathbf{\Gamma} : \mathbb{R}^n \to \mathbb{R}^6$ is **not square** (typically $n > 6$) $\Rightarrow$ redundancy.

Two paradigms

- **Cascade** (control + allocation): modular, easy to tune,
- **Unified** (NMPC over actuators): exploits over-actuation, handles delays / saturation natively.

f-MRAV control: architectures

Control structure

- **High-level controller** generates desired wrench:

$$\mathbf{w}_d = \begin{bmatrix} \mathbf{F}_d \\ \boldsymbol{\tau}_d \end{bmatrix} \in \mathbb{R}^6$$

- **Low-level allocation:** compute rotor inputs

$$\mathbf{u} = [\omega_1^2, \dots, \omega_n^2]^\top \quad \text{s.t.} \quad \boldsymbol{\Gamma}\mathbf{u} = \mathbf{w}_d$$

Geometric control

$$\mathbf{F}_d = m(\ddot{\mathbf{r}}_d + K_p \mathbf{e}_p + K_v \mathbf{e}_v + \mathbf{g})$$
$$\boldsymbol{\tau}_d = -K_R \mathbf{e}_R - K_\omega \mathbf{e}_\omega + \text{FF terms}$$

Allows full SE(3) tracking: arbitrary force & orientation.

Input allocation strategies

Objective: Solve for inputs $\mathbf{u}$ given desired wrench $\mathbf{w}_d$

$$\min_{\mathbf{u}} \; \|\mathbf{u}\|^2 \quad \text{s.t.} \quad \boldsymbol{\Gamma}\mathbf{u} = \mathbf{w}_d, \; \mathbf{u}_{\min} \le \mathbf{u} \le \mathbf{u}_{\max}$$

Pseudo-inverse (least-norm solution); **QP-based allocation** (handles constraints); **Nonlinear optimization** if $\boldsymbol{\Gamma}$ is configuration-dependent.

Practical notes

- $\text{rank}(\boldsymbol{\Gamma}) = 6$ for full wrench authority,
- Rotor $\hat{\mathbf{z}}_i$ and $\mathbf{r}_i$ determine control effectiveness,
- Actuator saturation and dynamics must be considered in real-time control.

Full-pose geometric control on SE(3)

Decoupled tracking of position and orientation

Reference trajectory $(\mathbf{r}_d(t), \mathbf{R}_d(t)) \in \mathbb{R}^3 \times \mathrm{SO}(3)$. With errors

$$\mathbf{e}_p = \mathbf{r} - \mathbf{r}_d, \qquad \mathbf{e}_R = \tfrac{1}{2}(\mathbf{R}_d^\top \mathbf{R} - \mathbf{R}^\top \mathbf{R}_d)^\vee, \qquad \mathbf{e}_\zeta = \boldsymbol{\zeta} - \mathbf{R}^\top \mathbf{R}_d \boldsymbol{\zeta}_d,$$

the desired wrench expressed in the **world frame** is

$$\mathbf{F}_d^{\mathcal{W}} = m(\ddot{\mathbf{r}}_d + \mathbf{g}) - \mathbf{K}_p \mathbf{e}_p - \mathbf{K}_v \mathbf{e}_v, \qquad \boldsymbol{\tau}_d^{\mathcal{B}} = -\mathbf{K}_R \mathbf{e}_R - \mathbf{K}_\zeta \mathbf{e}_\zeta + \boldsymbol{\zeta} \times \mathbf{J}\boldsymbol{\zeta} + \mathrm{FF}.$$

Then express force in the body frame: $\mathbf{F}_d^{\mathcal{B}} = \mathbf{R}^\top \mathbf{F}_d^{\mathcal{W}}$ – **no projection on $\hat{\mathbf{b}}_3$ needed**.

Properties

- Almost-global **exponential** pose tracking,
- Recovers the SE(3) controller of Lee *et al.* as a special case when only $\hat{\mathbf{b}}_3$ thrust is feasible,
- Naturally degrades to position-only tracking when the reference pose is infeasible.

Laterally Bounded Fully-Actuated (LBFA)

Most real f-MRAVs have a **cone-shaped** feasible force set: strong thrust along $\hat{\mathbf{b}}_3$, weaker lateral force. $\Rightarrow$ controller projects $\mathbf{F}_d^{\mathcal{B}}$ on the admissible set before allocation.

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Thank you

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